applied optics

High-precision tilt sensor using a folded Mach–Zehnder geometry in-phase and quadrature interferometer

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A new high-sensitivity homodyne in-phase and quadrature (I/Q) -interferometer scheme for measuring the tilt change of a target is presented. The new tilt sensor is a Mach–Zehnder interferometer folded by the target, in which the phase change is induced by the in-plane tilt change of the target but is not sensitive to any other motions. The interferometer is specially designed to minimize interferences caused by environmental perturbations. The induced phase is directly measured by using the I/Q-demodulation scheme. The tilt sensor exhibits an excellent sensitivity 10 prad/Hz$^{1/2}$ at a frequency slightly above 1 Hz and a 0.4 prad/Hz$^{1/2}$ at a frequency higher than 30 Hz.

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1. INTRODUCTION

An optical lever scheme has been widely used for many tilt sensor applications in both fundamental physics and precision engineering because it can provide a high sensitivity with a relatively simple optical arrangement. It has been extensively used for measuring the small deflection angle of a cantilever in a scanning probe microscope, initial alignments of the mirrors in a gravitational wave detector, and so forth [1–5]. However, in some advanced applications, it requires a higher sensitivity, a wider dynamic range, and lower drift tilt measurements than those obtained by using the optical lever. An extremely high-sensitivity tilt sensor scheme is required for measuring the twist angle of a torsion pendulum [6,7] and the angular displacement between two orthogonal test-mass bars [2] in research related to gravitation. Therefore, there has been extensive research on developing an extremely high-resolution tilt sensor schemes. Very high-sensitivity tilt sensors utilizing various Sagnac interferometer schemes in conjunction with a weak signal amplification have been proposed by many authors [8–10]. For example, Turner *et al.* showed that less than 10 prad/Hz$^{1/2}$ in the 10–200 Hz band can be measured [8].

The purpose of our present work is to develop an interferometer tilt sensor (ITS) that can be used for the initial alignments of mirrors in KAGRA, a cryogenic gravitational wave antenna being installed in the Kamioka mine in Japan [11]. The separation between two mirrors of the Fabry–Perot cavity in one arm of KAGRA is 3 km, and the beam size at the mirror is 3 cm. Therefore, in order to hold the alignment for a long period of time, the long-term drift of the ITS must be lower than 10 μrad during the measurement interval. In addition, the sensitivity required for locking mirrors is 0.1 μrad higher than 0.1 Hz [12]. Although a properly designed and engineered optical lever can meet the requirements for the initial mirror alignments for KAGRA, employing a higher sensitivity, a wider dynamic range, and a lower drift tilt sensor is important because not only does it make the initial alignment procedure easier, but it can also keep the alignment for a long period of time.

In this paper, we introduce a new ITS that can provide extremely high sensitivity, low drift, and wide dynamic range measurements. Geometrically, the ITS is a Mach–Zehnder interferometer (MZI) folded by the target mirror. Two arms of the ITS, however, are orthogonally polarized with respect to each other so that a homodyne in-phase and quadrature-demodulation (I/Q-D) scheme in which the phase difference between two arms can be directly measured without any calibration is employed [13,14]. Therefore, if the separation between the two beams is known, the tilt angle can be directly obtained from the corresponding phase value by a simple calculation. It will be shown that the folded MZI (FMZI) scheme is not sensitive to any other motion of the target but the tilt about the axis perpendicular to the incident plane, which allows us to measure the true tilt of the target. Moreover, it will also be shown that the input and output optics are inherently immune to environmental perturbations. As a result, the ITS can provide excellent sensitivity, 0.4 prad/Hz$^{1/2}$ at above 30 Hz, which is more than a 10 dB improvement over the Sagnac interferometer scheme [8]. To the best of our knowledge, it is the first time that such a FMZI scheme has been applied to the tilt measurement.

2. EXPERIMENTAL SETUP

A schematic of the optical arrangement of the ITS is shown in Fig. 1. A commercial 1.2 mW single- frequency, stabilized He–Ne laser (Newport Co. R-39727) is used as the light source, and a 57 dB optical isolator, OI, is inserted into the beam path to block any stray light reflected back into the laser. The plane of polarization of the incident beam is oriented at 45° from the preferred axes of the polarizing beam splitter, PBS1, and split into two orthogonally polarized beams with equal amplitudes. The reflected beam from PBS1 is aligned properly by using the right angle prism, RP1, to make it parallel to the transmitted beam from PBS1. PBS1 and RP1 are firmly mounted together and referred to as the input optics. Two output beams from the input optics are reflected at the target mirror TM and recombined by using RP2 and PBS2, which are firmly mounted together and referred to as the output optics. We found that in the later part of our work, a commercial lateral beam splitter (Edmund Optics, 47-190) can be used as the input and output optics.

The preferred axes of PBS2 are aligned parallel to the plane of the polarizations of the corresponding incident beams so that the two orthogonally polarized beams are combined but not mixed at PBS2. If the beams are aligned properly, then the combined beams at the PBS are collinear and have exactly the same optical path lengths. If the target mirror is tilted in the plane of incidence by a small angle, $\delta\theta$, then it can be shown that the optical path length difference (OPLD) between two beams is given by

$$\delta l = 2d\delta\theta, \tag{1}$$

where d is the spacing between the two beams. The concomitant phase difference $\Delta\phi$ induced by the tilt can be measured directly by using the I/Q-demodulator (DR) shown inside the dotted rectangle in Fig. 1. The I/Q-DR consists of two balanced mixers: each one has a half-wave plate (HWP), PBS, and two photodiodes (PDs). The HWP is used for rotating the plane of polarization by 45° so that the two beams are mixed at the PBS. A quarter-wave plate is used for adding a 90° phase difference between the two orthogonally polarized beams of the corresponding balanced mixer. Therefore, the balanced detectors are identical but have a 90° phase difference with respect to each other. If the output signal from one balanced mixer, say v_I, is proportional to cos $\Delta\phi$, then the output signal from the other port, v_Q, is proportional to sin $\Delta\phi$. Therefore, the tilt angle can be measured directly by using the following relationship:

$$\tan^{-1}\left(\frac{v_Q}{v_I}\right) = \Delta\phi = \frac{2\pi\delta l}{\lambda} = \frac{4\pi d\delta\theta}{\lambda}, \tag{2}$$

where λ is the wavelength of the light source. Due to imperfect optical components and PDs, v_I and v_Q may have different amplitudes. This imbalance can be minimized by using a proper amplifier. Moreover, it can be shown that a small amplitude difference has negligible contribution to Eq. (2). It can also be shown that the phase measurement is independent from the intensity change of the light source and, thereby, any correlated relative intensity noise in the light source can be rejected.

The induced phase can be directly measured without any calibration procedure by processing the output signals from the I/Q-DR. Since the tangent function has a large slope everywhere, the sensitivity for measuring the small phase change is always high. As mentioned earlier, it does not require any feedback control to keep the optimum sensitivity for the phase or tilt measurements. In order to calculate the tilt angle, however, the spacing between the two beams must be given. The distance was measured by using a scanning photodiode with a 100 μm mounted pinhole. The photodiode is mounted on a linear motion stage moving across the direction perpendicular to the two beams. The beam separation can also be deduced from the tilt measurement results obtained by using a calibrated tilt stage.

As shown in Fig. 2(a), any displacement of the TM does not change the OPLD between the two beams of the FMZI. The intensity reached at the PDs, however, may be varied by the

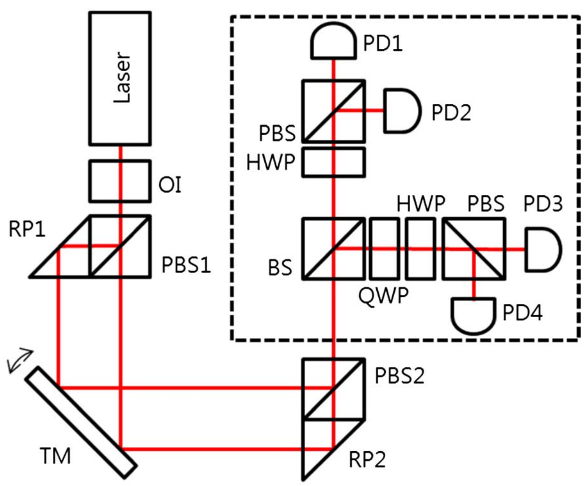

Fig. 1. Experimental arrangement. The I/Q-D arrangement is shown in the dotted rectangle. The arrow shows the tilt direction to which the FMZI is sensitive.

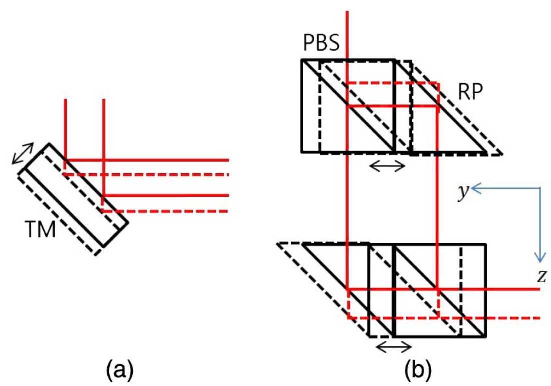

Fig. 2. Changes in optical path lengths of the two beams before (solid lines) and after (dashed lines) the displacements of (a) the TM and (b) the input and output optics.

displacement because of the beam position change at the PD caused by the displacement. However, since the I/Q-D scheme can measure the phase change regardless of the intensity change, the true tilt change of the TM can be measured. It can also be shown geometrically that the FMZI is only sensitive to in-plane tilt or yaw, but insensitive to pitch of the TM. Moreover, as shown in Fig. 2(b), any displacements and pitch of the input and output optics do not affect the corresponding OPLDs. Another advantage, which will be shown experimentally as a demonstration, is that the interferences caused by beam pointing fluctuations can be minimized in the FMZI scheme.

3. MEASUREMENT RESULTS AND DISCUSSION

The performance of the FMZI scheme is tested by using a mirror mounted on a piezoelectric transducer (PZT)-driven tilt stage (Thorlabs, KC1-T-PZ). A small tilt is deliberately added to the stage at 2 Hz, and the frequency spectra of the measurement results are shown as the black trace in Fig. 3. The tilt angle is calculated by using Eq. (2) from the measured phase value, and the result is in good agreement with the applied tilt. Since the PZT-driven tilt stage is not stable enough for evaluating the performance of the ITS, a better tilt stage (Thorlabs, POLARIS K1F6) was used for measuring the spectral noise characteristics. The results are shown as the gray trace (blue online) in Fig. 3. It is reasonable to claim that based on the noise floor measurement results, the minimum tilt that can be measured by using the FMZI scheme is 10 prad/Hz$^{1/2}$ at slightly above 1 Hz and 0.4 prad/Hz$^{1/2}$ at a frequency higher than 30 Hz, which is close to the photon noise limit, ~0.28 prad/Hz$^{1/2}$ for a 1 mW He–Ne laser. The sensitivity is lower than the photon noise limit especially at a low frequency because of the phase noise induced by environmental perturbations such as the vibrations of optical components, acoustic noise, atmospheric turbulence, and so forth. As we can see in Eq. (1), in principle, the sensitivity can be improved by increasing the separation between the two beams. The larger the beam separation, however, the more the FMZI becomes susceptible to environmental perturbations. Therefore, the FMZI must be installed in a very well-controlled environment, a vacuum, for example, for a large separation application.

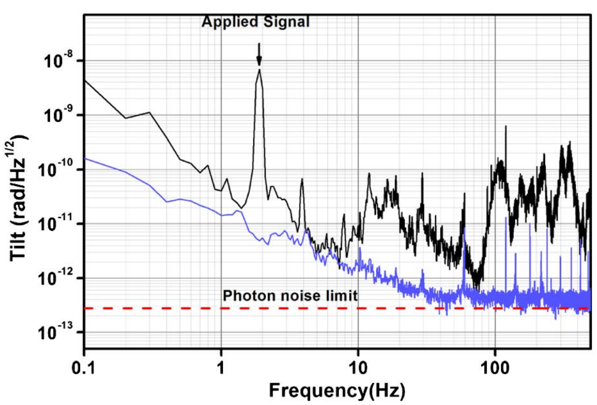

Fig. 3. Measurement results: the black trace is the measurement result when a tilt is applied to the PZT-driven tilt stage, and the gray (blue online) trace represents the noise floor measurement using a stable tilt stage.

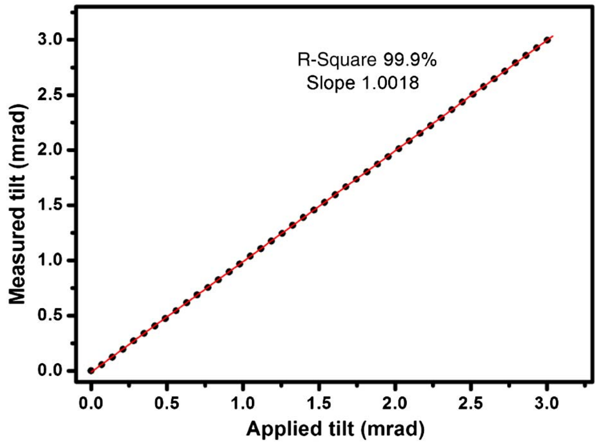

Fig. 4. Measured tilt angles at various applied tilt angles (solid circles) and the best fit to the linear function.

The tilt angles were measured at various applied tilts. The measurement results are plotted as solid dots and their best fit is represented as the solid line in Fig. 4, which shows the excellent linearity of the FMZI scheme as predicted in Eq. (2). It is obvious from this measurement result that the FMZI scheme can measure the true tilt of the TM but is insensitive to any other movements of the TM.

As mentioned earlier, it can be shown that the interferometer is not sensitive to any movements and/or beam pointing fluctuations of the laser. To prove this immunity, the beam pointing direction is sinusoidally changed at 45 Hz, as shown in Fig. 5, by using a folding mirror, FM, mounted on the PZT tilt stage. The measured tilt angle for 96 μrad of the beam pointing direction change is 1.6 μrad, which is evidence of the immunity of the FMZI scheme.

The ITS is designed to be used for the initial tilt alignments of the test masses, the main interferometer mirrors, of KAGRA. At the present moment, because of the cryogenic environment, both the input and output optics must be placed outside the corresponding vacuum chamber, where the test masses are installed. The total length of the two beams between the input and output optics is approximately 2 m. In order to evaluate the

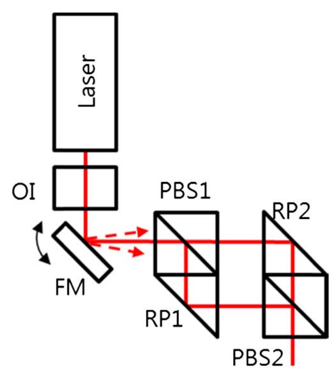

Fig. 5. Experimental arrangement for evaluating the immunity of the beam pointing error.

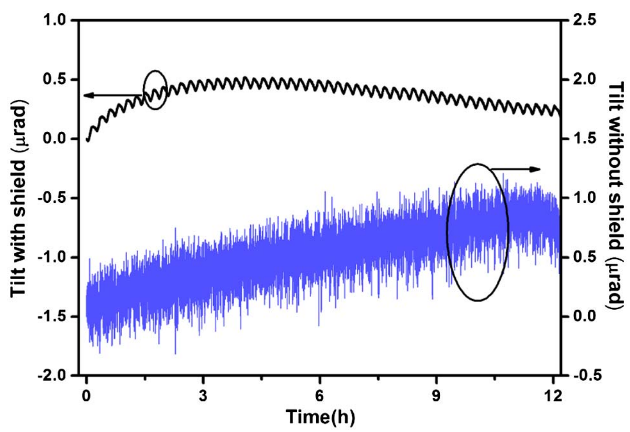

Fig. 6. Twelve hour long, long-term phase measurements for the target mounted on the fixed tilt stage. The gray (blue online) and the black traces represent the results with and without the shield, respectively.

performance of the proposed FMZI as the ITS, we set up the 2 m path length interferometer on the optical table. The room temperature is controlled at 23 °C with ±0.5°C fluctuations. The phase measurement result in an open-air environment for 12 h is shown as the gray (blue online) trace in Fig. 6. It shows a relatively large phase noise, which may be dominantly coming from atmospheric turbulence. The phase noise is reduced significantly when the two beams of the interferometer are placed inside a homemade shield made by a PVC pipe. The measurement results with the shield are shown as the black trace in Fig. 6, which shows a significant reduction of the phase noise. We believe that the small amplitude ripples on the trace result from the on–off switching of the temperature controller for the lab.

It must be emphasized that in our application, for the initial tilt alignment of the mirrors in the gravitational wave antenna, the input and output optics will be mounted right next to the vacuum chamber so that most of the probe beam paths will be located inside of the vacuum chamber [12]. The environment is much more hospitable than that in our lab; therefore,

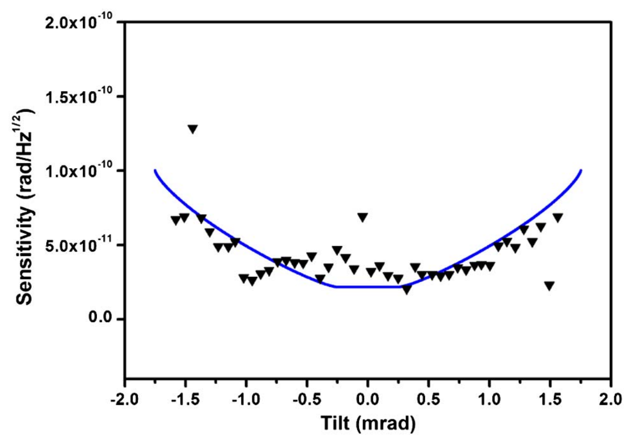

Fig. 7. Measurement results for angle-dependent sensitivity are shown as solid triangles, and the solid (blue online) line is a theoretical plot.

interferences caused by environmental perturbations may be smaller than that measured in our present work. The drift that occurred during 12 h-long tilt measurement is 0.52 μrad. Another 12 h-long measurement with the back-to-back arrangement, where the input and output optics are placed as close as possible without the folding mirror, resulted in an equivalent drift of 14.7 nrad. The equivalent tilt is deduced from the phase measurement result by using Eq. (2). Therefore, we can say that the drift may mainly result from the tilt stage used in the current experiment. Both the long-term drift and resolution for the tilt measurements very well satisfy the requirements for the initial mirror alignment.

In our current arrangement, the measurement range, another important parameter of the tilt sensor, is limited by the size of the active area of the photodiodes, which are 3 mm in diameter, and small misalignments between the photodiodes. The measured sensitivities at various tilt angles are shown in Fig. 7. In these measurements, the noise floors are estimated from the frequency spectrum by averaging the phase values or equivalent tilts in between 40 and 45 Hz and are regarded as the sensitivities at the corresponding tilt angles. The angle-dependent sensitivity is theoretically calculated by using the following assumptions: (1) The received optical power is proportional to the overlapping area between the optical beam and the active area of the photodiode, and (2) the sensitivity is proportional to the square root of the received optical power, i.e., the overlapping area. The theoretical plot represented by the blue curve shows a good agreement with measurement results. The results show that the FMZI scheme can provide a good sensitivity of up to ±1.5 mrad. It should be mentioned that the dynamic range can be increased by using a wider-area photodiode and/or a focusing lens in front of each photodiode.

4. CONCLUSIONS

In summary, a novel interferometer tilt sensor scheme using the FMZI arrangement has been presented. Geometrically, an FMZI arrangement has been used to provide two balanced optical paths for interferometric tilt measurements. The phase change induced by the corresponding tilt was directly measured by using the homodyne I/Q-D scheme. It has been shown that this arrangement can provide an excellent performance in tilt measurements: ∼0.1 nrad/Hz$^{1/2}$ resolution at 0.1 Hz, 0.52 μrad (14.7 nrad in the back-to-back arrangement) drift in 12 h, and a ±1.5 mrad measurement range, both of which meet the requirements for the initial mirror alignments for KAGRA, ∼1 nrad/Hz$^{1/2}$ resolution at 1 Hz, 1 μrad drift, and 1.3 mrad measurement range. It should be noted that in the case of using a conventional optical lever scheme, in order to meet the requirements, a measured value was fitted to an empirical equation to compensate for the temperature dependence and drift of the measurement system [12]. Since the compensation coefficients may subject to vary with the installing conditions for the device, it may require calibrations during the installation procedure. The ITS using the FMZI scheme, however, does not require any calibrations, because it is less sensitive to environmental conditions. The FMZI is only sensitive to the in-plane tilt measurement. Since both the pitch and yaw must be measured for the initial alignment of a test mass, two FMZIs must be employed. We are now in the process of designing a prototype in which two sets of input and output optics are correspondingly integrated into one input and output port of the ITS for the pitch and yaw measurements. We also would like to mention that our new ITS scheme has many other applications in high-precision engineering, such as aligning and/or evaluating fabrication equipment for ultra-high precision machining, semi-conductors, and other nano or sub-nano fabrications. In principle, since the resolution can be improved by increasing the separation between two beams, a similar arrangement can also be used for reading an extremely small angular displacement of the torsion pendulum or torsion bar. A higher-power laser can also be used for increasing the signal-to-noise ratio. All the optical components must be securely mounted and placed in a vacuum environment for these applications because the interferometer will be more susceptible to environmental perturbations for large beam separations.

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