Dilution factor of the pendulum

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The mechanical loss of a pendulum suspension, $\phi_{\rm pend}$ can be shown to be [1]

$$\phi_{\rm pend} = \frac{k_{\rm fiber}}{k_{\rm fiber} + k_{\rm gravity}}\,\phi_{\rm fiber} \sim \frac{k_{\rm fiber}}{k_{\rm gravity}}\,\phi_{\rm fiber}$$

$$D \equiv \frac{k_{\rm fiber}}{k_{\rm gravity}} = \frac{\sqrt{T\,Y\,I}}{2\,l^2} \Big/ \frac{mg}{l} = \frac{\sqrt{T\,Y\,I}}{2mg\,l}$$

The ratio of the gravitational spring constant $k_{\rm gravity}$ to the elastic spring constant of the suspension fibre $k_{\rm fiber}$ is the pendulum dilution factor D. Here T, Y, I, m, g, l are wire tension, Young modulus, moment of cross section, mass which is suspended by wire, gravity constant and wire length, respectivelly.

$$T = mg$$

$$I = \frac{\pi\,d^4}{64}$$

Here d is wire diameter.

$$\therefore\ D = \frac{d^2}{16\,l}\sqrt{\frac{\pi\,Y}{mg}}$$

Table 1: Pure Al Wire Case

		CLIO	KEK
Young modulus	Y	70 GPa	
Mass	m	0.450 kg	0.150 kg
Wire diameter	d	0.5 mm	
Wire length	l	0.400 m	0.100 m
Number of wires		4	1
Dilution factor	D	8.7e-3	6.0e-2

References

[1] P. R. Saulson, Physical Review D 42, 2437 (1990).