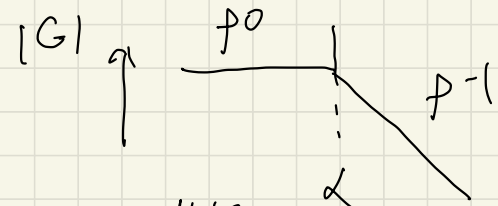
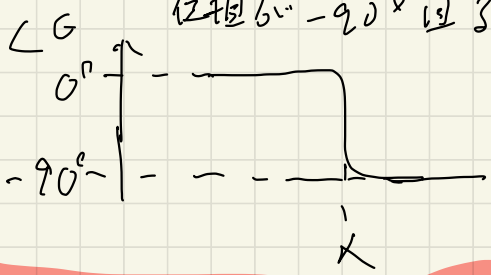
$$G(s) = \frac{(s+\beta_1)(s+\beta_2)\times\cdots\times}{(s+\alpha_1)(s+\alpha_2)\times\cdots}$$

α : pole

ゲインが $(\times p^{-1})$ される

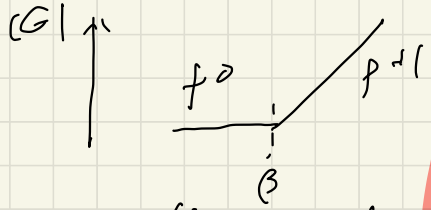


位相が $-90°$ 回る

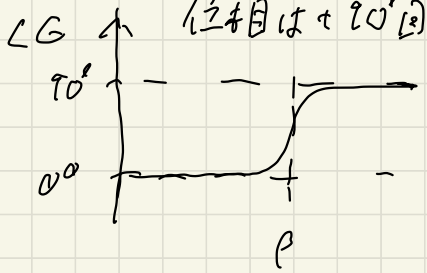


β : zero

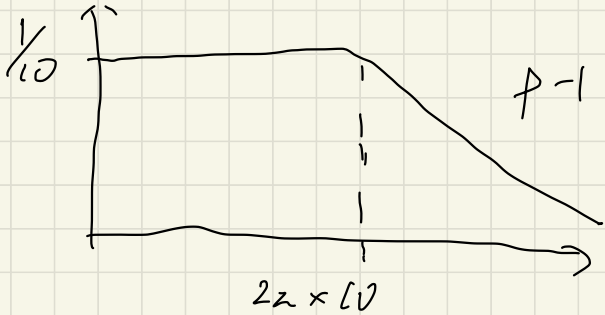
ゲインが $(\times p^{+1})$ される



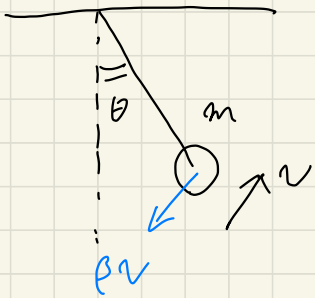
位相は $+90°$ 回る.



ex) $G(s) = \dfrac{1}{s+10}$



☆ 9/9　抵抗がある場合のふり子の伝達関数を考えよう.



運動方程式

$$m\frac{d^2x}{dt^2} = -mg\frac{x-x_0}{l} - \beta\frac{dx}{dt}$$

ラプラス変換

$$\mathcal{L}[x(t)] = X(s),\qquad s^2X = -\frac{g}{l}(X-X_0) - \frac{\beta}{m}\cdot sX$$

$$\left(s^2 + \frac{\beta}{m}s + \frac{g}{l}\right)X = \frac{g}{l}X_0$$

$$\frac{X}{X_0} = \frac{g/l}{s^2 + \frac{\beta}{m}s + \frac{g}{l}}$$

$$s = i\omega,\quad \sqrt{\frac{g}{l}} = \omega_0\quad \frac{\beta}{m} = 2\gamma\omega_0$$

$$\frac{X}{X_0} = \frac{\omega_0^2}{-\omega^2 + 2\gamma\omega_0 i\omega + \omega_0^2}$$

$$= \frac{\omega_0^2(\omega_0^2 - \omega^2 - 2\gamma\omega_0 i\omega)}{(\omega_0-\omega)^2 + (2\gamma\omega_0\omega)^2}$$

$$G = \left|\frac{X}{X_0}\right| = \sqrt{\frac{\omega_0^4}{(\omega_0^2-\omega^2)^2+(2\gamma\omega_0\omega)^2}}$$

$$\angle G = \arg\left(\frac{X}{X_0}\right) = -\tan^{-1}\left(\frac{2\gamma\omega_0\omega}{\omega_0^2-\omega^2}\right)$$

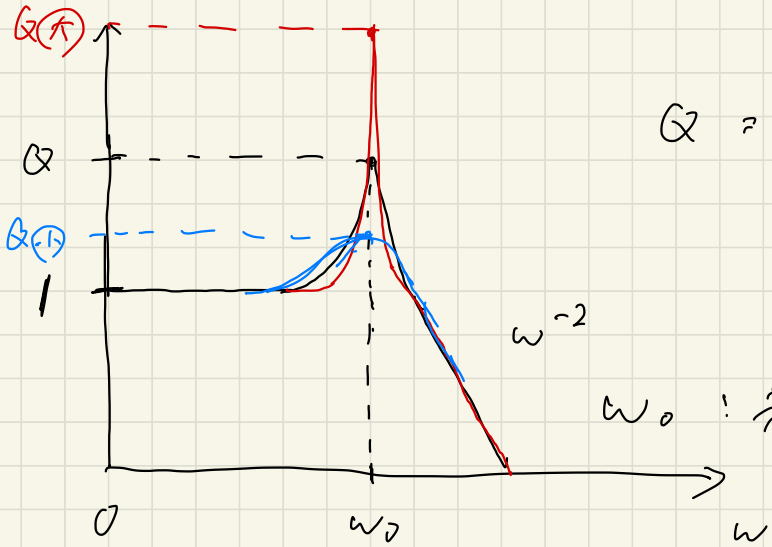
$$G(\omega=\omega_0) = \sqrt{\frac{\omega_0^4}{0+(2\gamma\omega_0^2)^2}} = \frac{1}{2\gamma} = \frac{m}{\beta}\omega_0$$

$$\equiv Q$$

→ Q値

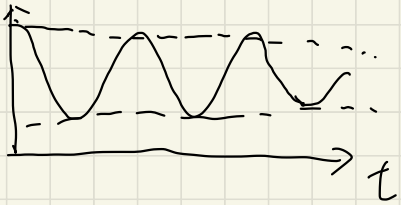
$$Q = \frac{m}{\beta}\omega_0$$

○ゲイン

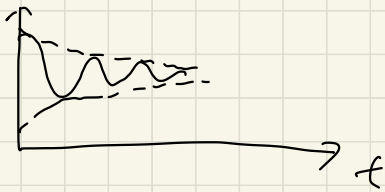


ω_0：共振周波数

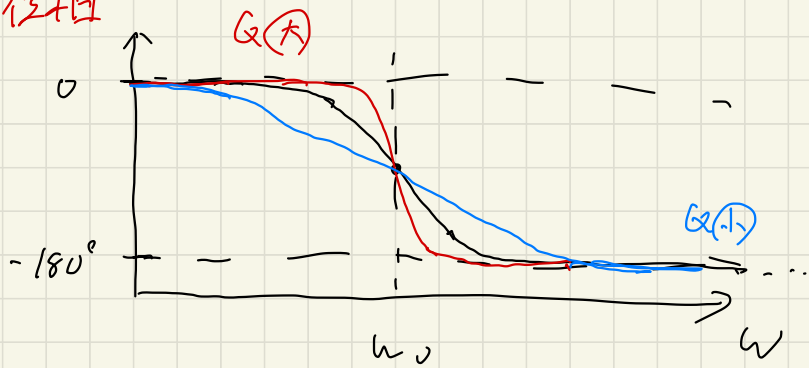
$\beta \to$ 小, $Q \to$ 大

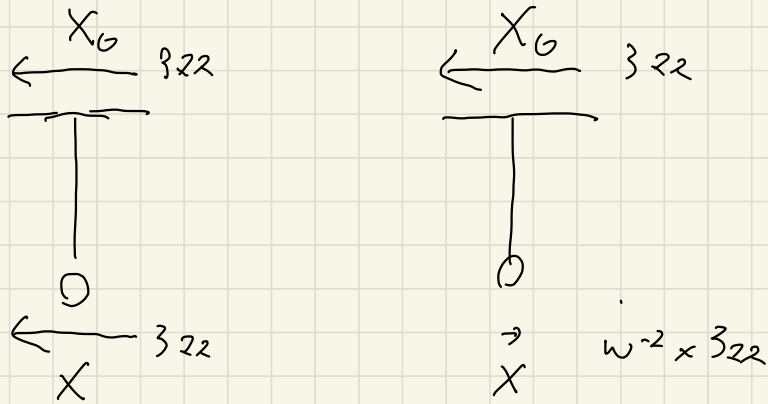


$\beta \to$ 大, $Q \to$ 小

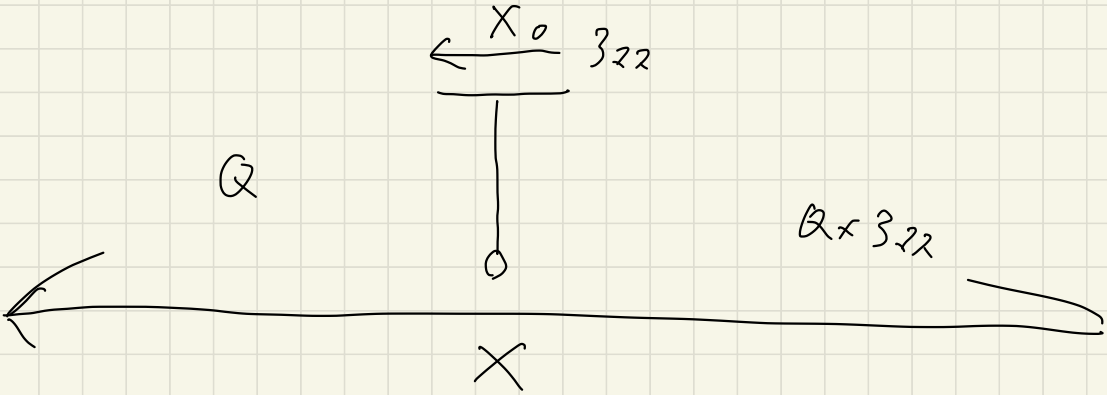


○ 位相

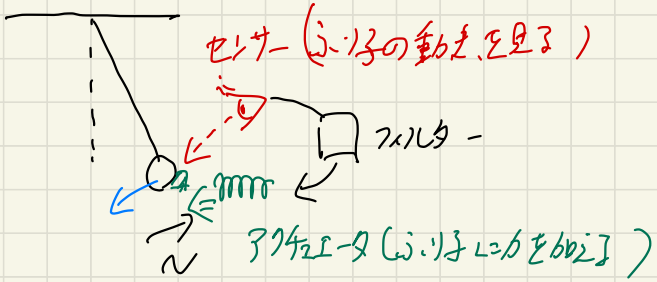




$\omega < \omega_0$ $\omega = \omega_0$ $\omega > \omega_0$

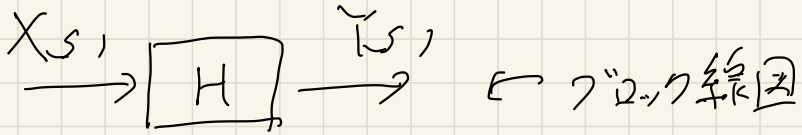


☆ フィードバックループの伝達関数.



ふり子の揺れを減衰させたい.

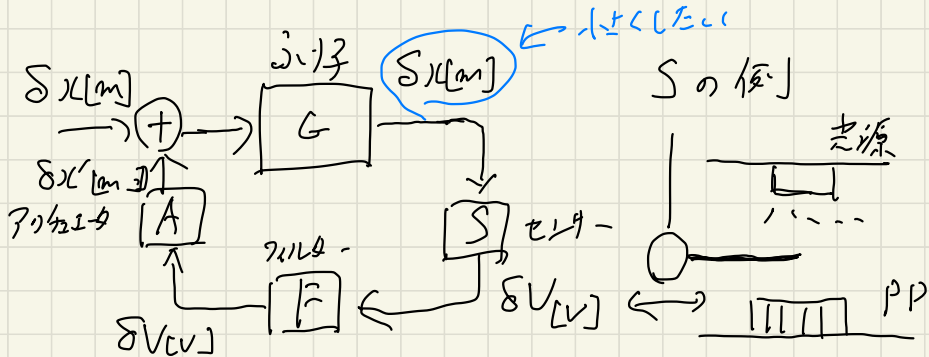
(復) ある線形システムをモデル化



伝達関数

$$H = \frac{Y(s)}{X(s)}$$

○ フィードバックループのブロック線図

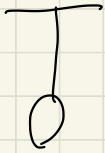


→ フィードバックループの周波数応答

$G \times S \times F \times A$

宿題

HW



まっすぐのふり子で、 $t = t_Q$

$t = 0$ のときから、Q回振動した後の

ふり子の揺れの大きさ.

$x(t)$

$$\frac{x(t_Q)}{x(0)}$$ を求めてみてください