Review of "BayesLine: Bayesian Inference for Spectral Estimation of Gravitational Wave Detector Noise" by Littenberg & Cornish, arXiv:1410.3852

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Abstract

Gaussian-noise modeling for power spectrum

- For detection confidence, signal characterization, model selection, parameter estimation
- Cubic splines (smoothly varying broad-band noise) + Cauchy (Lorentz) distribution (narrow-band line)
- Demonstrated on LIGO S5,S6 data

Introduction

LIGO/Virgo noise power spectrum

- Broad band: seismic (below 10 Hz), thermal from mirror suspensions and coating (10-200 Hz), photon shot noise (higher freq.)
- Narrow band: mirror suspensions, AC electrical supply, sinusoidal motion imparted on the mirrors calibration

For stationary and Gaussian noise, likelihood in matched filtering:

$$\log p(d|\theta) = -\int_0^\infty \frac{|\tilde{r}(f;\theta)|^2}{S_n(f)} df + const,$$

where the residual $\tilde{r}(f;\theta) = \tilde{d}(f) - \tilde{h}(f;\theta)$.

- For non-stationary, non-Gaussian noise, BayesWave (to be introduced by Hayama-san)

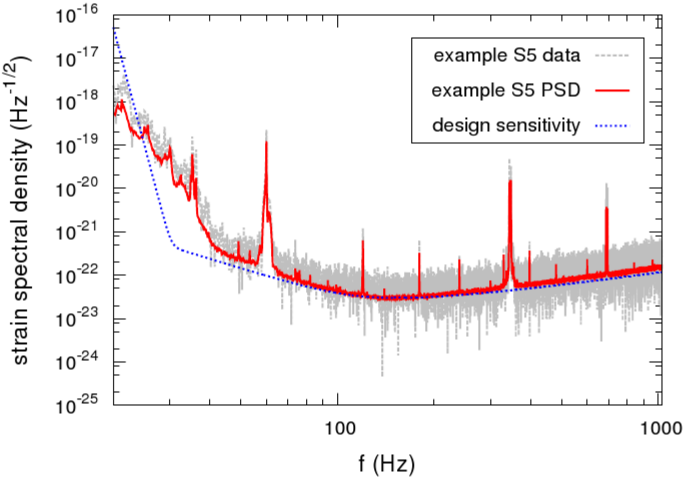
Introduction (cont.)

Why challenging?

- Past searches used a running average of the instrument power spectrum over long duration.
- But not stationary for times much longer than a few tens of second, depriving us of a sufficiently accurate reference noise spectrum.
- Sensitivity of detectors improves, particularly at low frequency. $\mathbf{1.4}\boldsymbol{M_{sol}}$ neutron star binary merger takes $\sim$ **25** sec to evolve from 40 Hz. Advanced LIGO/Virgo reach down to $\sim$ **10** Hz. The binary is in band $\sim \mathbf{10^3}$ sec, but reliance on averaging for PSD (power spectral density) estimation demands $\sim \mathbf{10^4}$ sec.

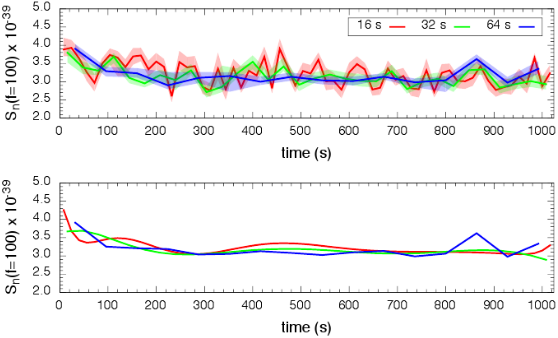
We want a parametrized model for the instrumental noise, just as we do for the signal, and the two will be deduced simultaneously during the parameter estimation.

The LIGO/Virgo Power Spectrum



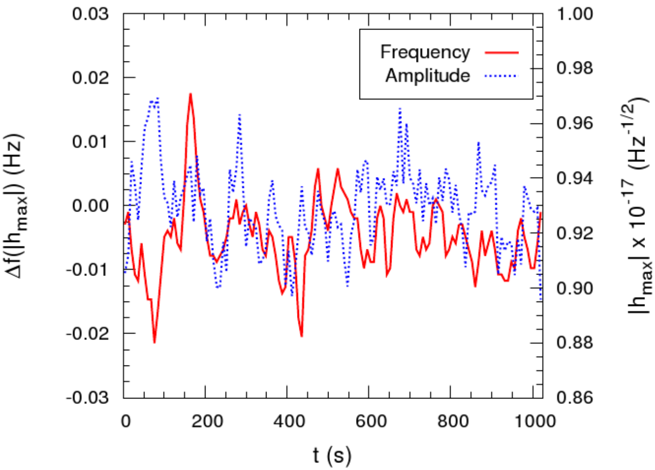
Red: used during actual parameter estimation follow-up analysis of a candidate chirp signal.

Non-stationarity of Noise



1024 sec of data which is the nominal duration used for PSD estimation in the follow-up of S6 triggers. Divide the data into 32(16,64) sec segments, the amount of data needed to parameter estimation of binary neutron star signal in S6.

Non-stationarity of Noise (cont)



60 Hz power line with 8 sec segments. Assuming the noise as being stationary over such long duration is not reliable.

The BAYESLINE method

MCMC (Markov chain Monte Carlo) is used to get posterior of PSD.

- the broad-band noise $S_S(f; \xi, N_S)$, where N_S is the number of control points in cubic spline $(\xi_i = (S_i, f_i))$:

$$S_{S,i}(f; \xi) = \sum_{k=1}^{3} c_k^{(i)} (f - f_i)^k, \qquad f \in [f_i, f_{i+1}], \qquad i = 0, 1, ..., N_S.$$

- the narrow-band noise: $S_L(f; \lambda; N_L)$, where N_L is the number of Cauchy distributions $(\lambda_j = (A_j, Q_j, f_j))$:

$$S_{L,j}(f; \lambda_j) = \frac{z(f) A_j f_j^4}{(f_j f)^2 + Q^2 (f_j^2 - f^2)}, \qquad j = 1, ..., N_L$$

  where $z(f)$ is introduced for truncation at $f_j \pm f_j/50$.

- Priors are Uniform (not described on A_j and Q_j).

The BAYESLINE method (cont.)

- Likelihood is as "matched filtering" with $S_n(f;\xi,N_S,\lambda,N_L) = S_S(f;\xi,N_S) + S_L(f;\lambda,N_L)$:

$$\log p(d|\xi,N_S,\lambda,N_L) = -\frac{2}{T}\sum_{f=1}^{N}\frac{|\tilde{d}(f)|^2}{S_n(f;\xi,N_S,\lambda,N_L)} + const.$$

- Because chain runs among models with different number of parameters, RJMCMC (reversible jump MCMC) is employed. Trans-model (different number of parameters) acceptance probability of $M_i \rightarrow M_j$ is

$$\min\left\{1, \frac{\pi_j(\theta_j)}{\pi_i(\theta_i)}\frac{\pi_{ji}q_{ji}(v_{ji})}{\pi_{ij}q_{ij}(u_{ij})}\left|\frac{\partial T_{ij}(\theta_i,u_{ij})}{\partial(\theta_j,u_{ij})}\right|\right\}, \qquad (\theta_j,v_{ji}) = T_{ij}(\theta_i,u_{ij}),$$

  where u_{ij} and v_{ji} are additional parameters and $\pi_i(\theta_i)$ is the target distribution (posterior).

- Marginalize over (ξ,N_S,λ,N_L) gives posterior PSD.

RJMCMC Green (1995) Biometrika 82: 711-732

Example (Green 1995)

Model 1: θ, model 2: (θ_1, θ_2). $\pi_{12} = \pi_{21} = 1$. Model 2 to 1, $\theta = (\theta_1 + \theta_2)/2$. Let $(\theta_1, \theta_2) = T_{12}(\theta, u) = (\theta - u, \theta + u)$ with $u \sim q_{12}$. The acceptance probability is

$$\min\left\{1, \frac{\pi_2(\theta_1, \theta_2)}{\pi_1(\theta)} \frac{1}{q_{12}(u)} 2\right\}.$$

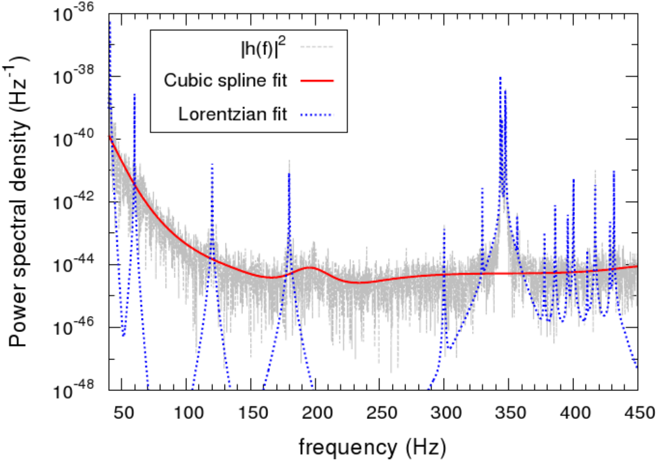
- For a bijection, "dimension matching", we need additional parameters.
- Reversibility (detailed balance):

$$\int_A \int_B K(x, dy)\pi(dx) = \int_B \int_A K(y, dx)\pi(dy)$$

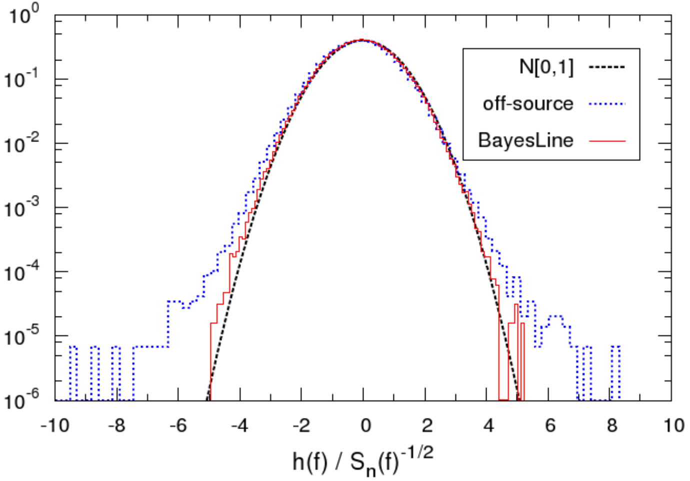
demands the acceptance probability.

Demonstration by LIGO S6

Requires 1 hour for 1024 sec, which is insignificant when compared to the computation cost of compact binary parameter estimation.

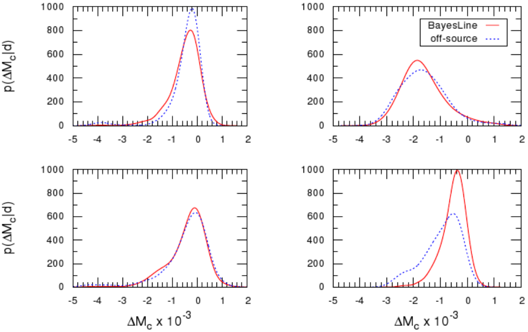


Demonstration by LIGO S6 (cont.)



Off-source PSD leaves behind significant large tails, i.e. incurs bias by the signal model in an attempt to account for the non-Gaussian residual.

Demonstration by LIGO S6 (cont.)



Simulated binary neutron star signals and use LALInference to estimate chirp mass. The posterior distribution.

In principle there is a risk that PSD model fit-out part of the signal. But in practice it is not, because the template provides a much better model for the signal than BayesLine.