Application of Non-linear Correlation Analysis on Environmental Monitors and Gravitational Wave Channel

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Contents

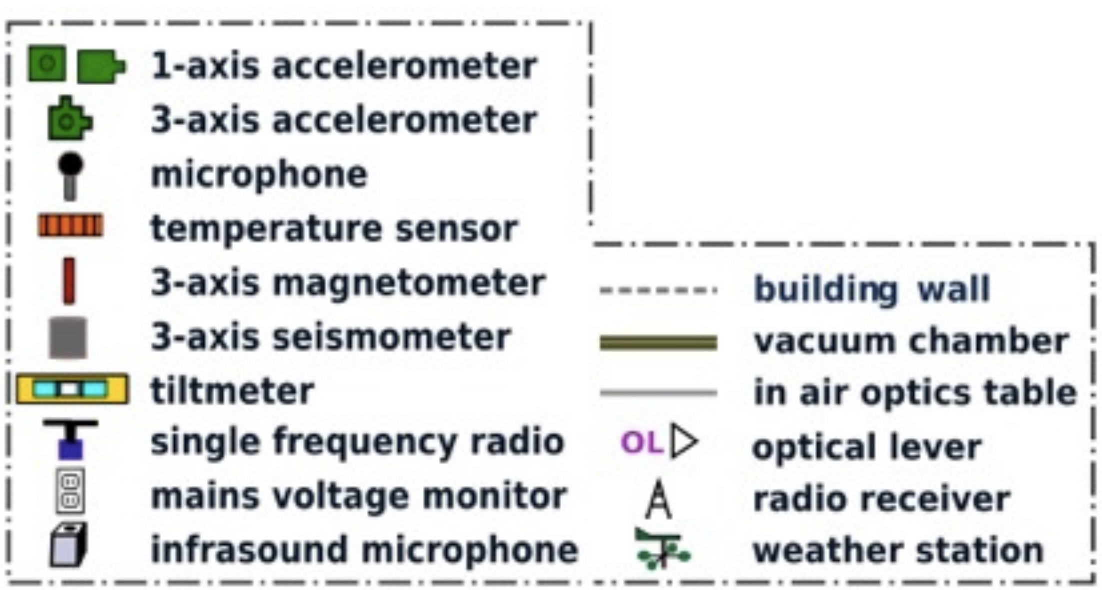
motivation - correlation analysis using environmental channels

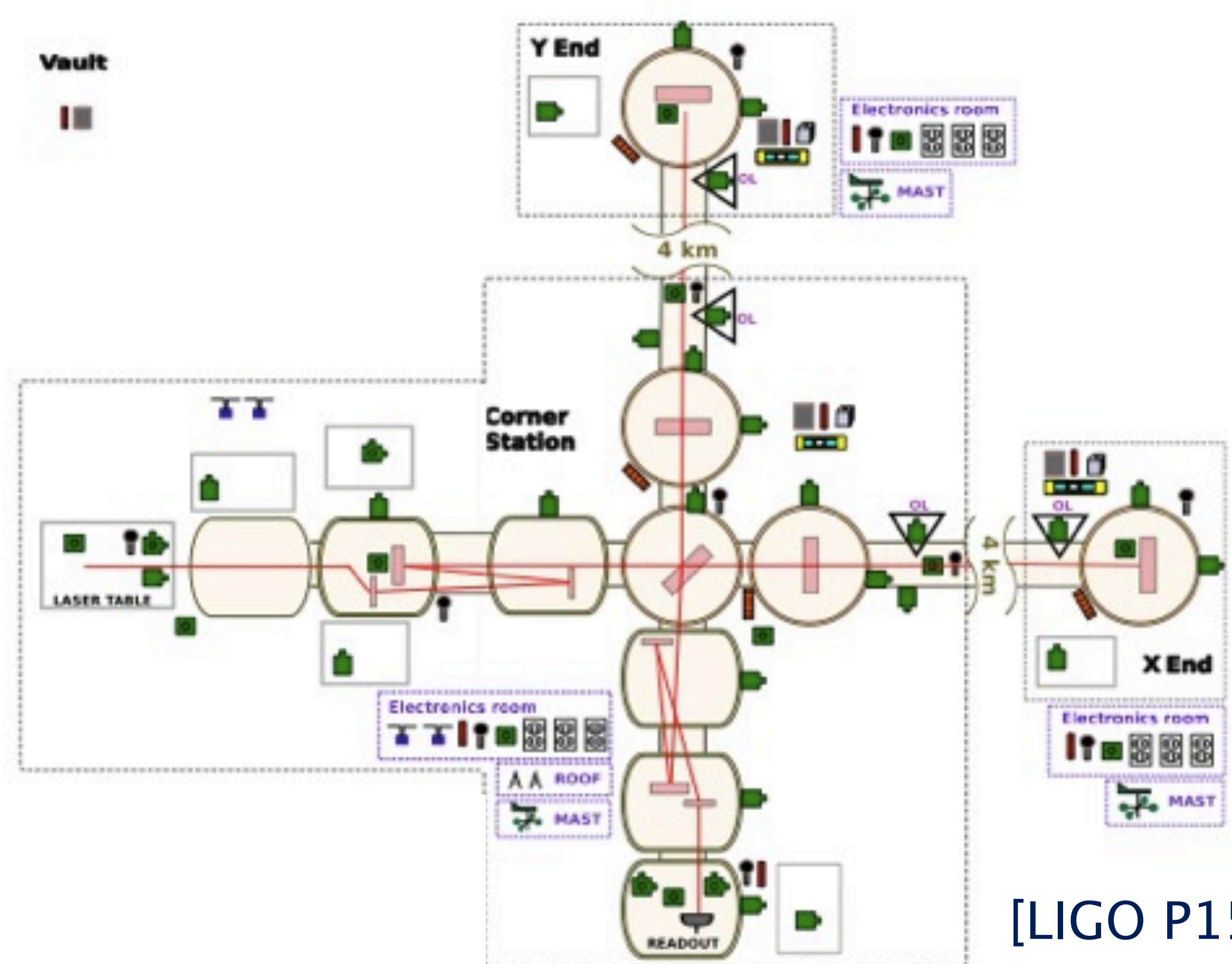
<u>Our Goal : find correlated noise between ~10000 physical environmental (PEM) channels and finally localize noise sources</u>

- Remove the noise sources to improve detector sensitivity
- Identify false trigger event generated by GW search pipeline

  -> the contribution to increase GW detection efficiency

In this talk,
- GW channel
  as sensitive channel to GW
- PEM channel such as,



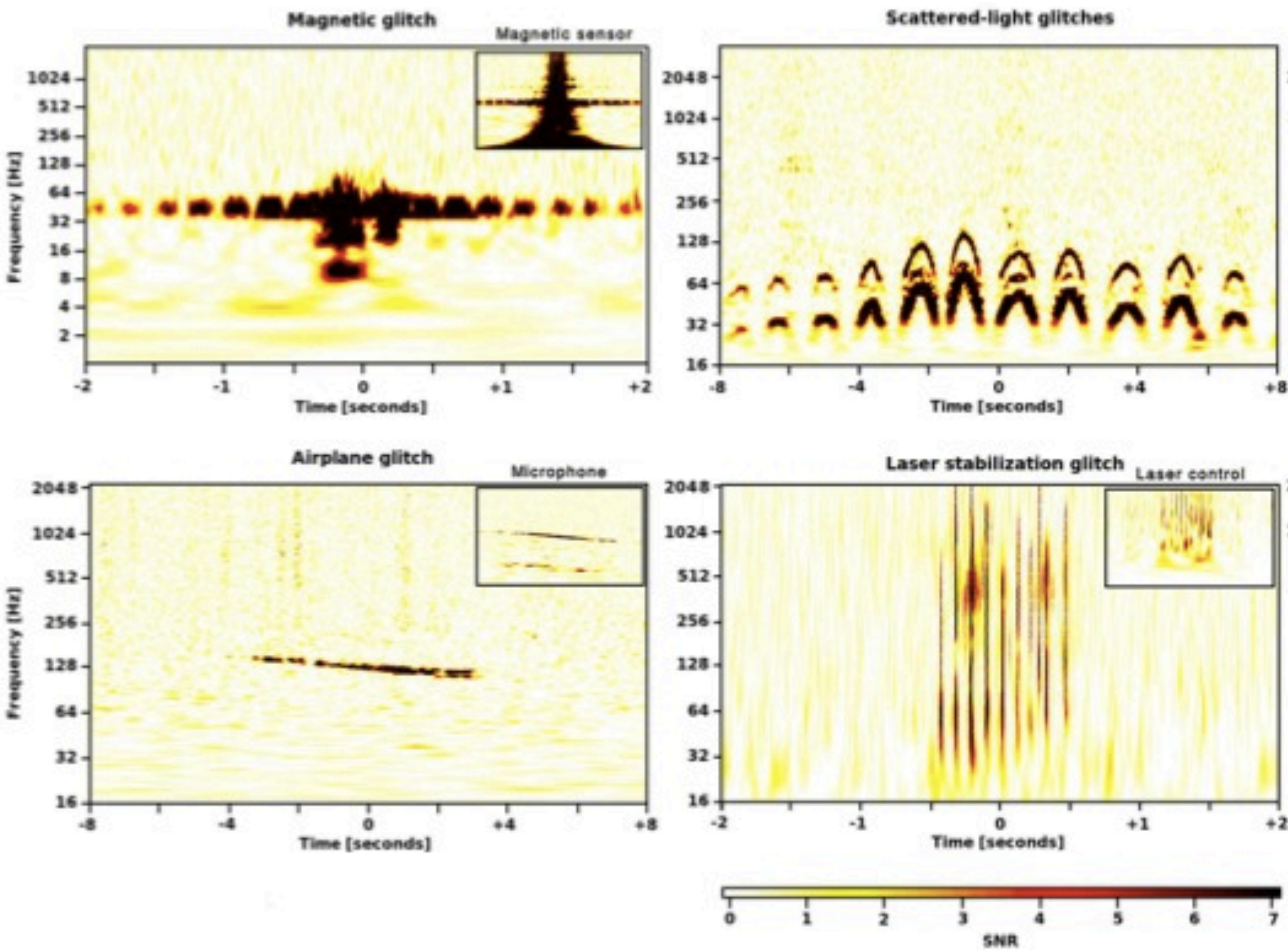


[LIGO P1500238]

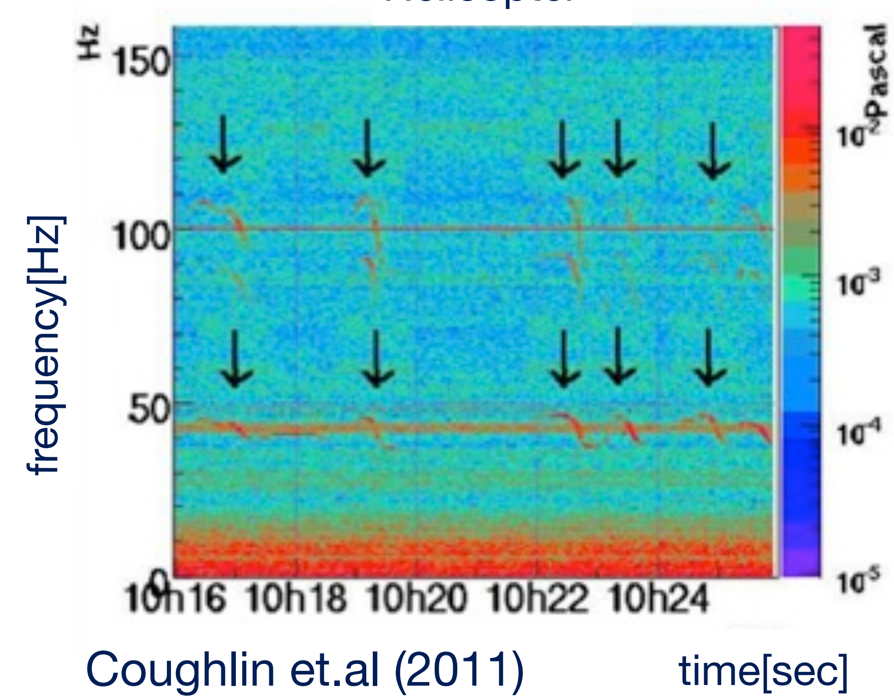
~10000 PEM sensors are installed in LIGO Livingston

Example of the correlated noise observed in LIGO and Virgo

J.Asis et al. (2012) [gr-qc 1203.5613]



Helicopter

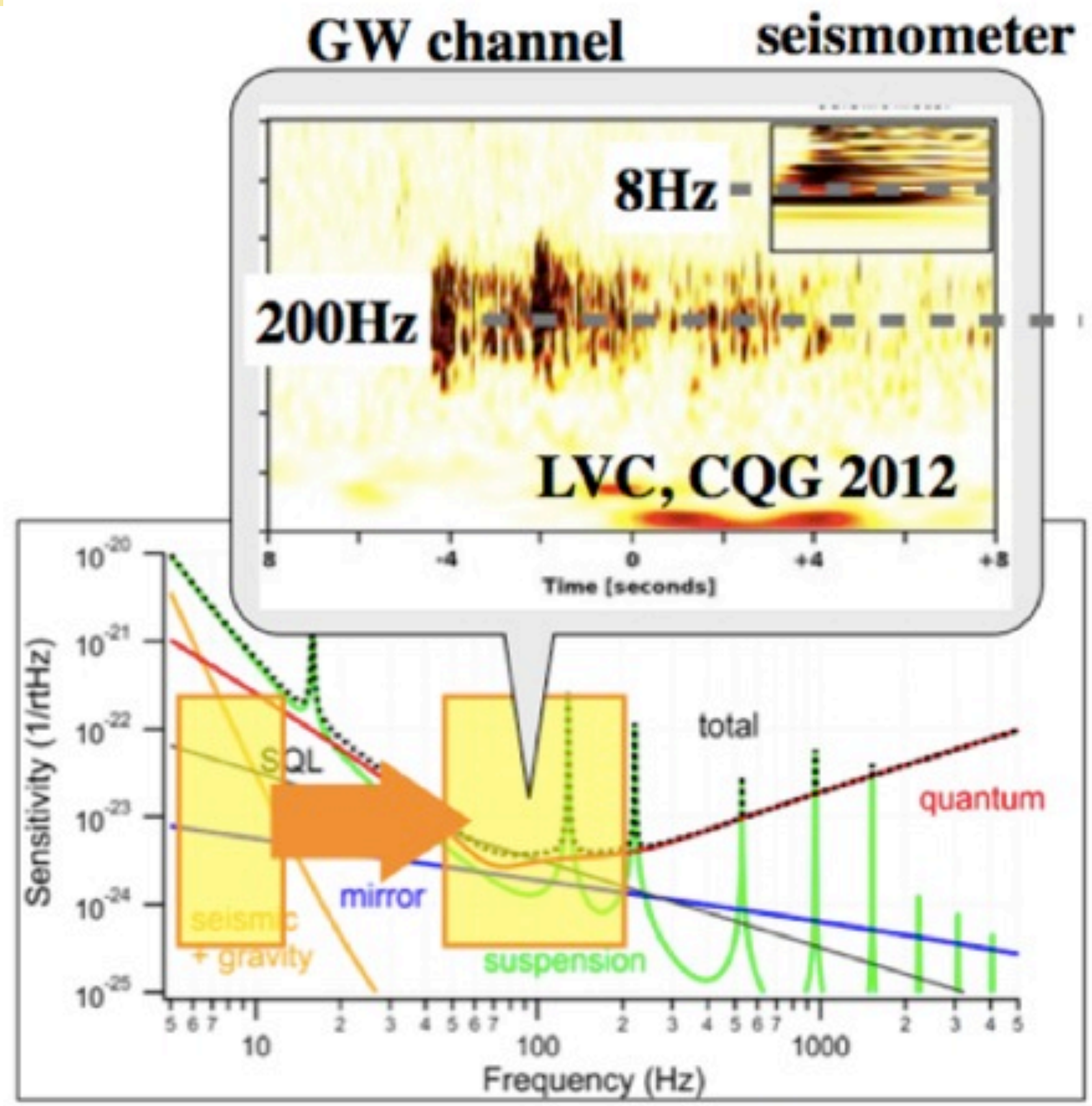


Coughlin et.al (2011)

*Example of **non-linear correlation** observed in LIGO and Virgo*

Up-converted noise:
Noise origin has low frequency component.
Transferred noise has high frequency component because of up-conversion.

Example:
Seismic glitches will cause scattered light noise.
Especially in bad weather day, seismic glitch is strong.
Non-linear correlation over a few Hz ~ a few hundreds Hz in GW channel was observed in detectors.



Sensitivity curve of KAGRA

Hayama (2014)
J.Asis et al. (2012) [gr-qc 1203.5613]
[CQG 27, 19 (2010) 194011]

To reveal noise source and noise contamination path, the detection of non-linear correlated noise is important.
The conventional correlation analysis methods can not detect the non-linear correlation.

Correlation analysis methods

In this study, two methods are used,

- Pearson Correlation Coefficient

  - efficient method to linear correlation $r = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}}$

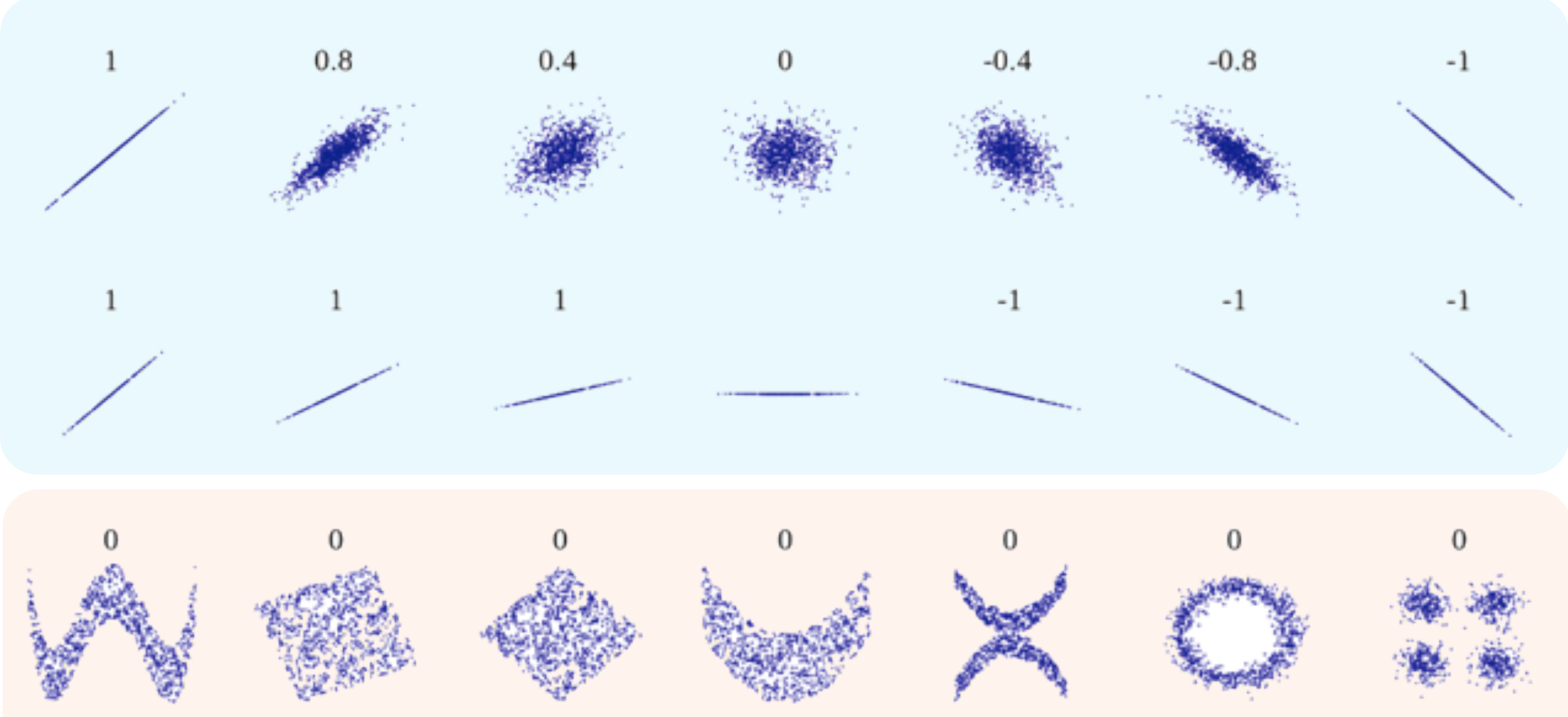
- **Maximum Information Coefficient (MIC)**

[David N. Reshef, et al. Science 334, 1518 (2011)]

linear correlation

Linear correlation →

non-linear correlation →



non-linear correlation

http://lectures.molgen.mpg.de/algsysbio12/MINEPresentation.pdf

Maximum Information Coefficient (MIC)

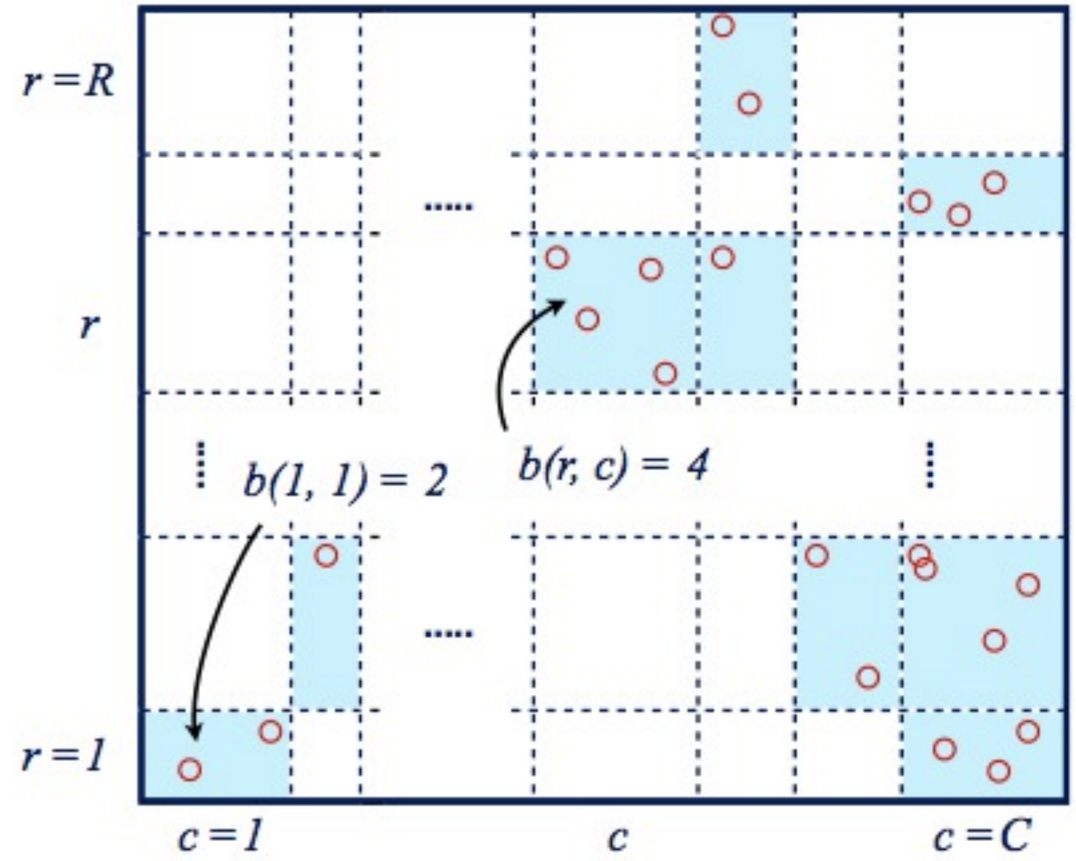
[David N. Reshef, et al. Science 334, 1518 (2011)]

o If a relationship exists between two data, a grid can be drawn on the scatter plot of two data that partitions the data to encapsulate that relationship.
o For each placement of partition, the mutual information is calculated.

$$I(R,C) = \sum_{r=1}^{R}\sum_{c=1}^{C} p(r,c)\log_2 \frac{p(r,c)}{p(r)p(c)},$$

p(r, c) : joint probability mass function
p(r), p(c) : marginal probability function

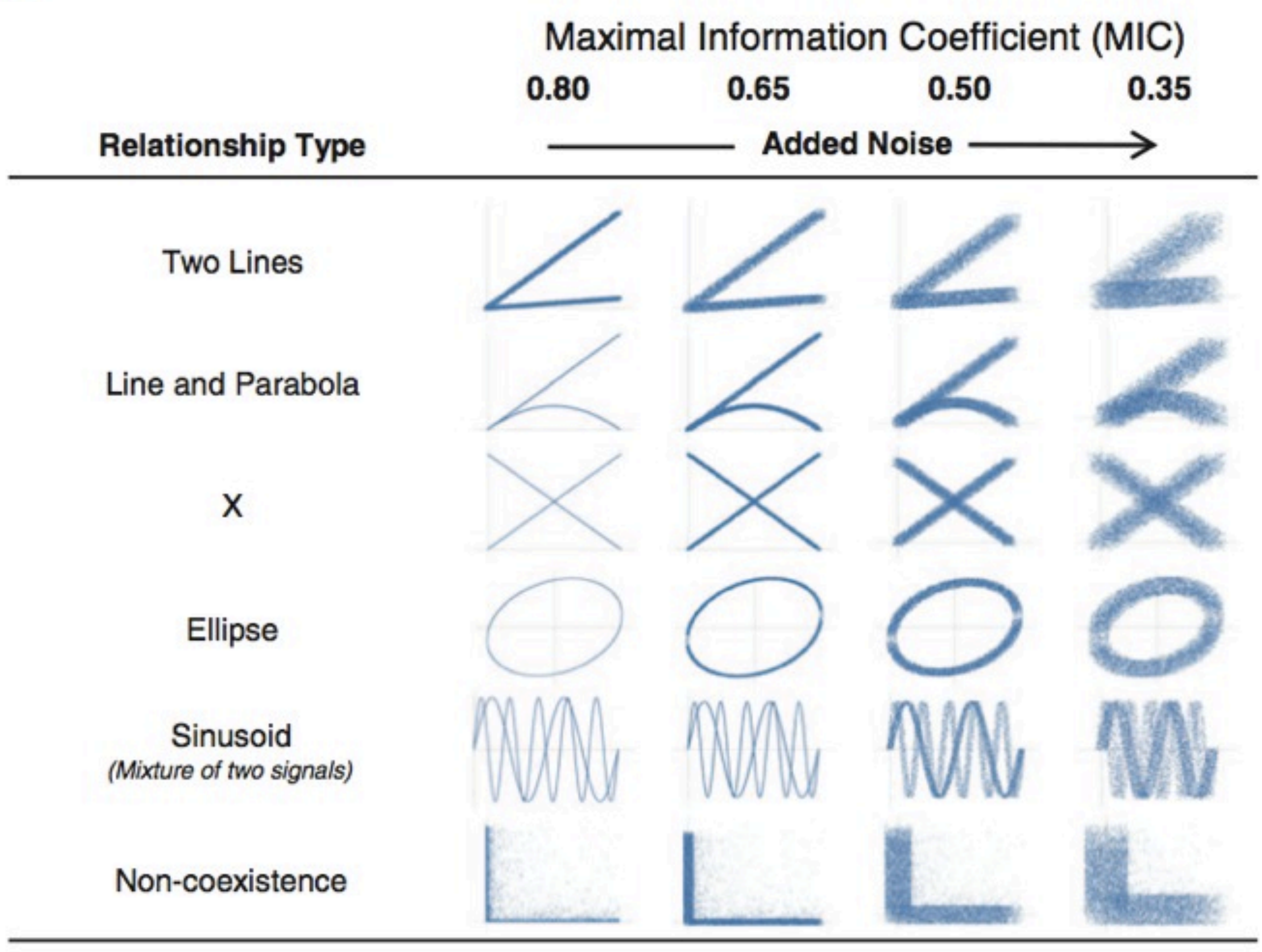
$$MIC(x,y) = \max_{RC<B(N)} \frac{I(R,C)}{\log_2(\min\{R,C\})},$$



o The MIC is defined as the mutual information maximized under all the possible grids with RC < B(N)
B(N) is maximal number of cell and is used $B(N) = N^{0.6}$ now.

Which correlation MIC can find?

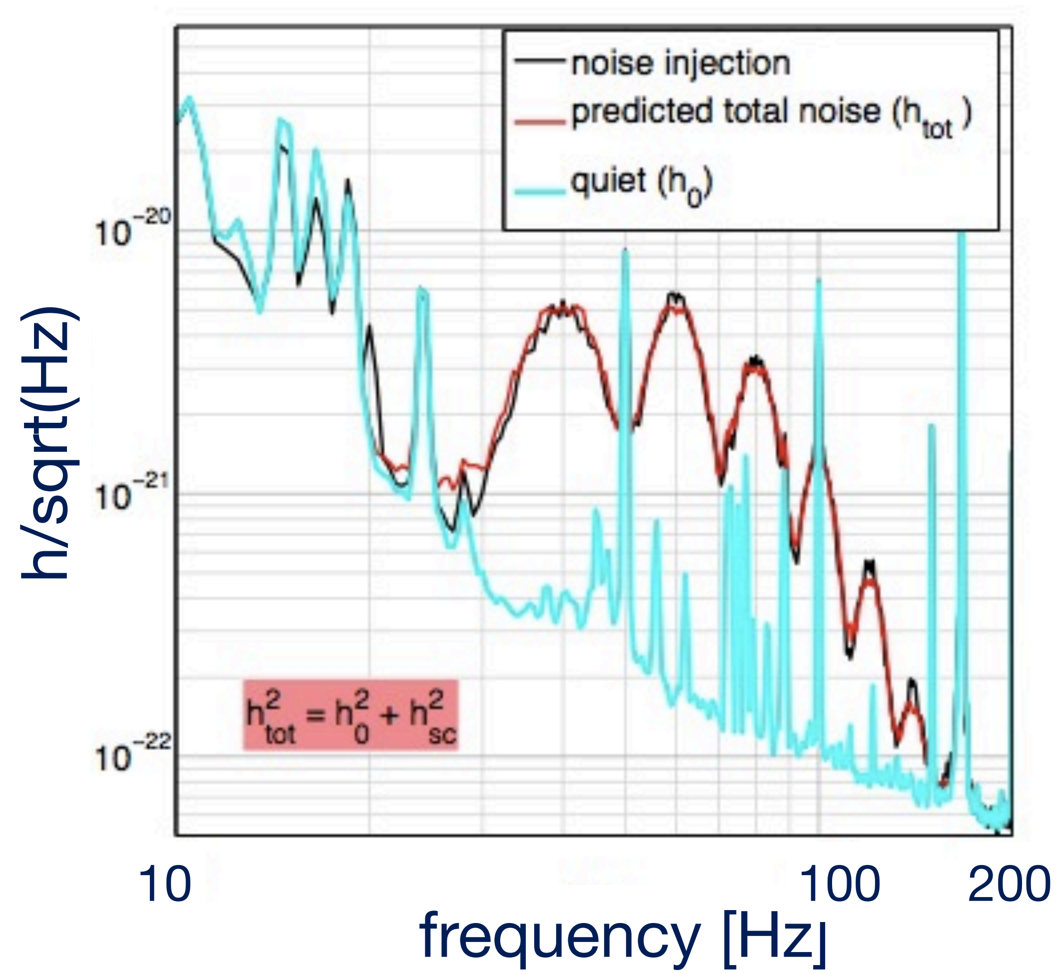
- MIC can find not only linear but also non-linear correlation.



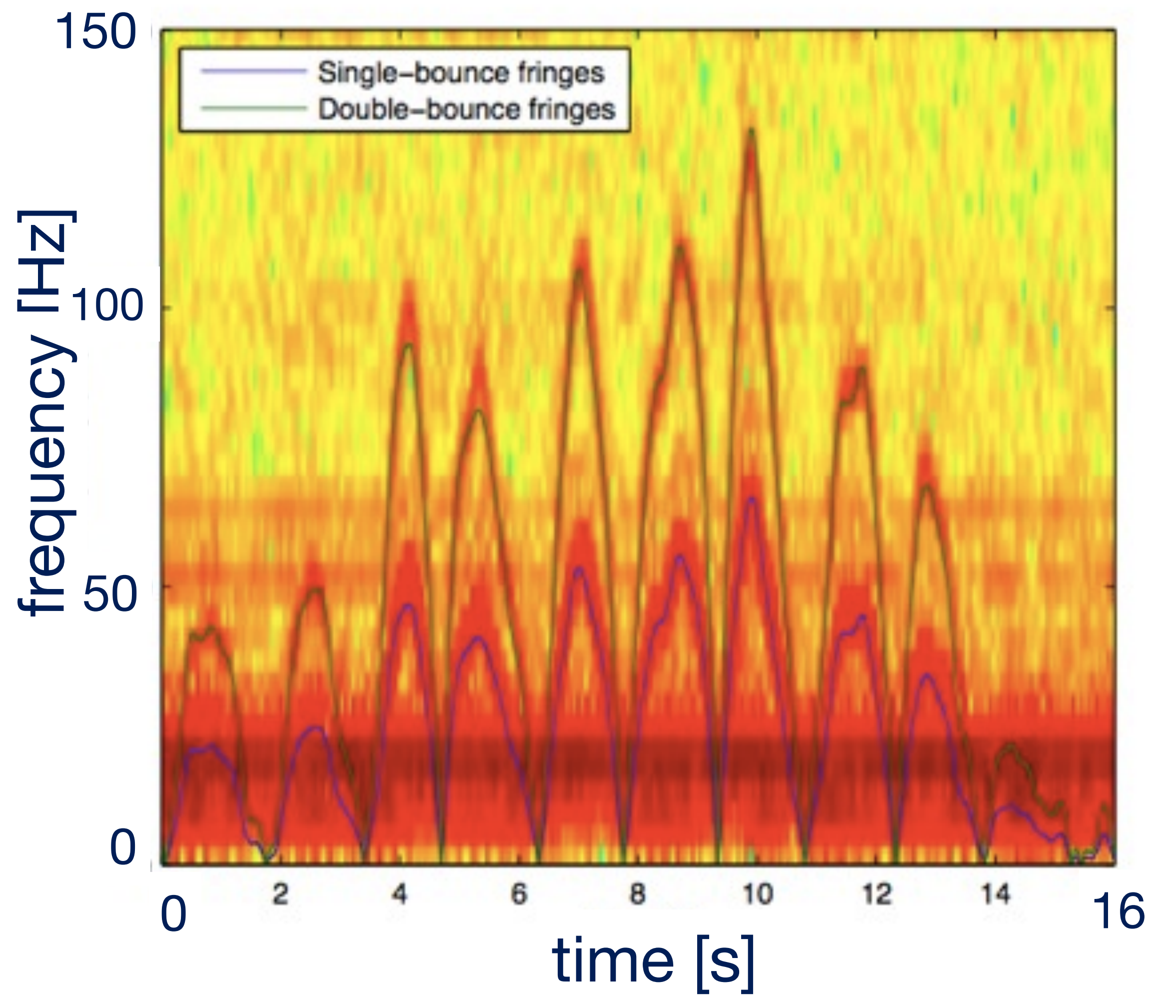
[David N. Reshef, et al. Science 334, 1518 (2011)]

Up-conversion noise observed at Virgo detector

[Classical and Quantum Gravity 27, 19 (2010) 194011]



The strong seismic activity generate the scattering light. The structure with many peaks are caused by scattering light noise. The contaminated sensitivity curve is worse more than 1 order.



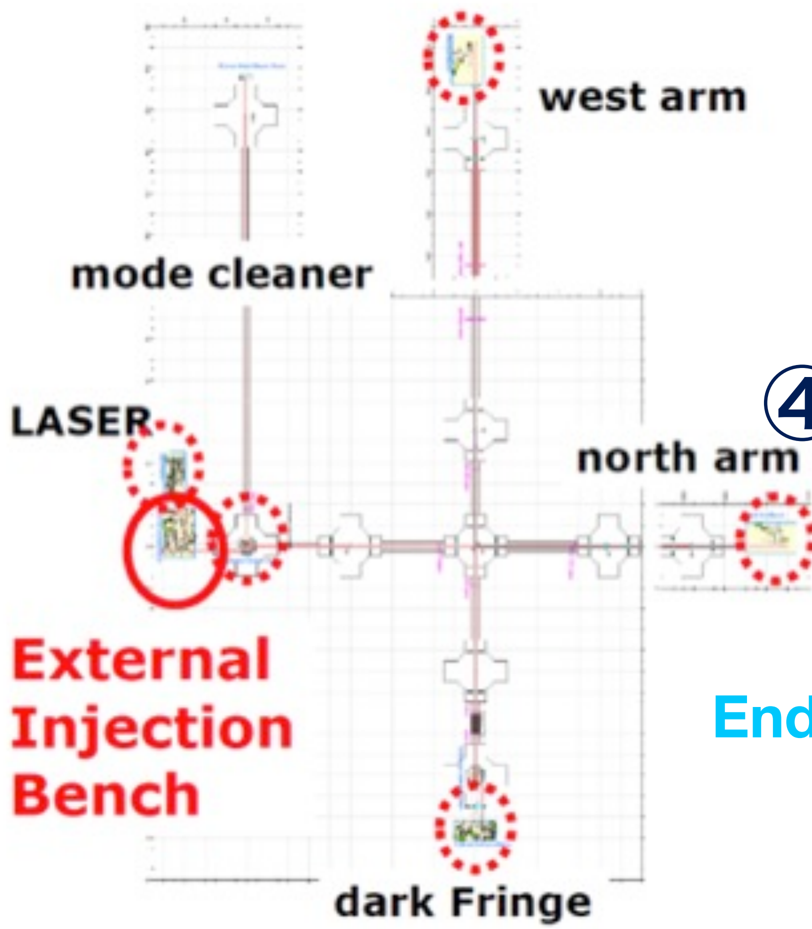
Spectrogram of secondary scattering light noise

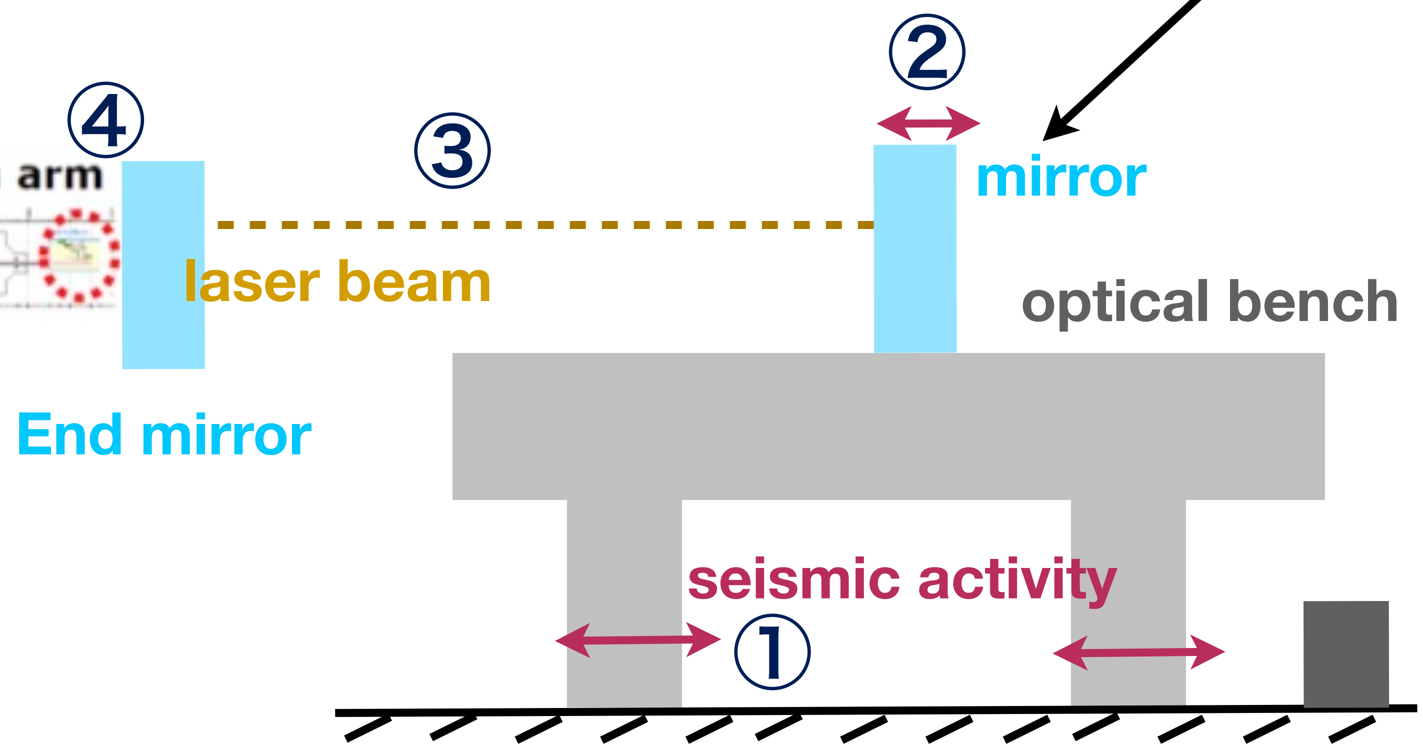
The past Virgo detector has been limited by this noise.
This up-conversion noise is solved now and well-modeled.

Mechanism of the up-conversion

[Classical and Quantum Gravity 27, 19 (2010) 194011]

We monitor motion of mirror via accelerometer. $\delta x_{sc}(t)$





microseism

We monitor GW channel $s(t)$ **.**

Up-conversion noise model observed at Virgo detector

[Classical and Quantum Gravity 27, 19 (2010) 194011]

up-conversion noise

$$h_{sc}(t) = G \cdot \sin\left(\frac{4\pi}{\lambda}(x_0 + \delta x_{sc(t)})\right)$$

x_0 : distance between end mirror and reflector

$\delta x_{sc}(t)$: displacement of mirror by seismic activity

G : constant factor depending on interferometer $(G = 5 \times 10^{-20})$

λ : laser wavelength (1064 [nm])

GW channel : $s(t) = h_{sc}(t) + n(t)$

n(t) : fundamental noise of GW detector
(Virgo sensitivity is used. Assuming gaussian and stationary noise)

Up-conversion noise model observed at Virgo detector

[Classical and Quantum Gravity 27, 19 (2010) 194011]

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GW channel : $s(t) = h_{sc}(t) + n(t)$

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(Virgo sensitivity is used. Assuming gaussian and stationary noise)

displacement of mirror excited by seismic activity

$$\delta x_{sc}(t) = A_m \sin(2\pi f_m t) \exp(-t/\tau) + n_{seis}(t)$$

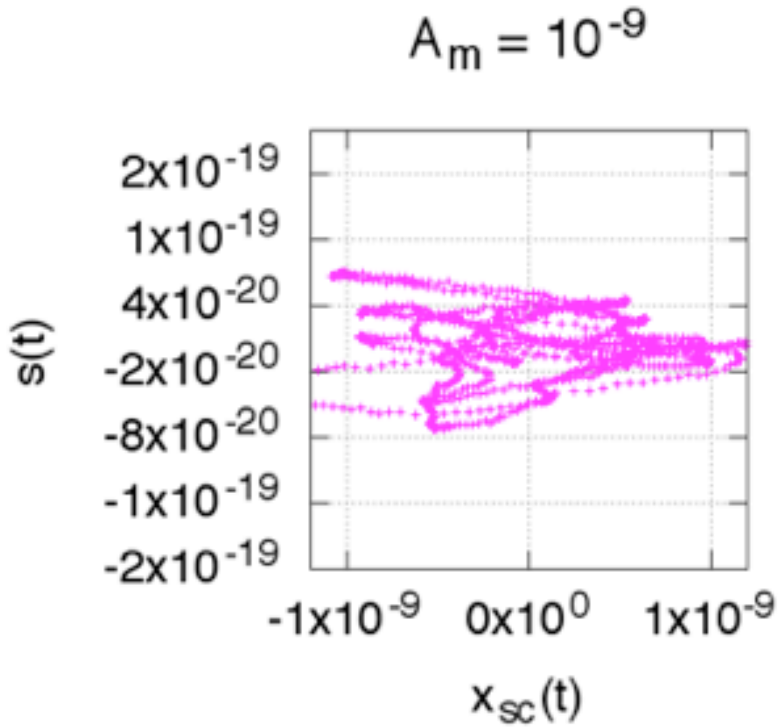
A_m : amplitude of mirror's displacement

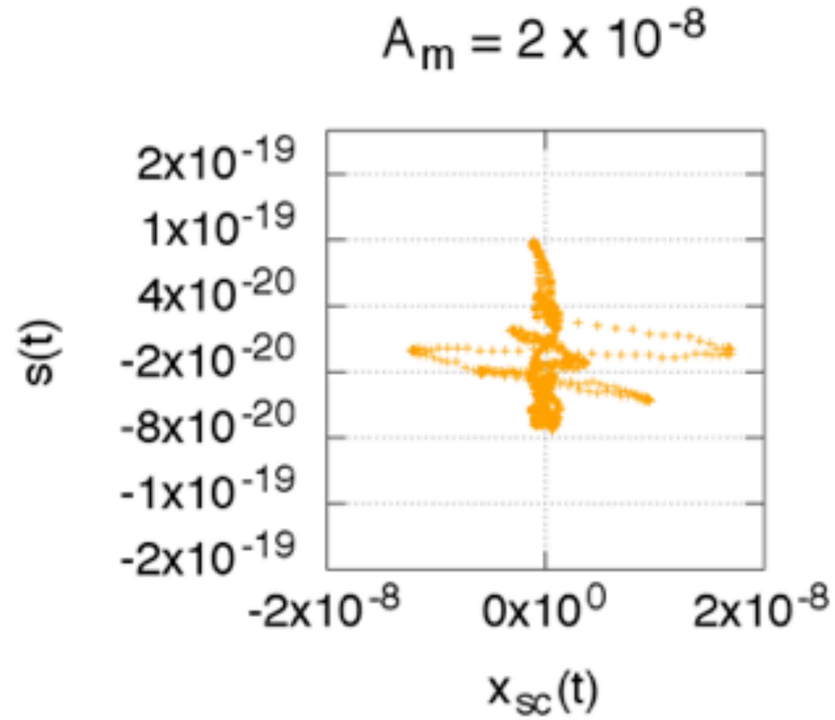
$f_m = 15[\mathrm{Hz}]$: resonant frequency of optical bench

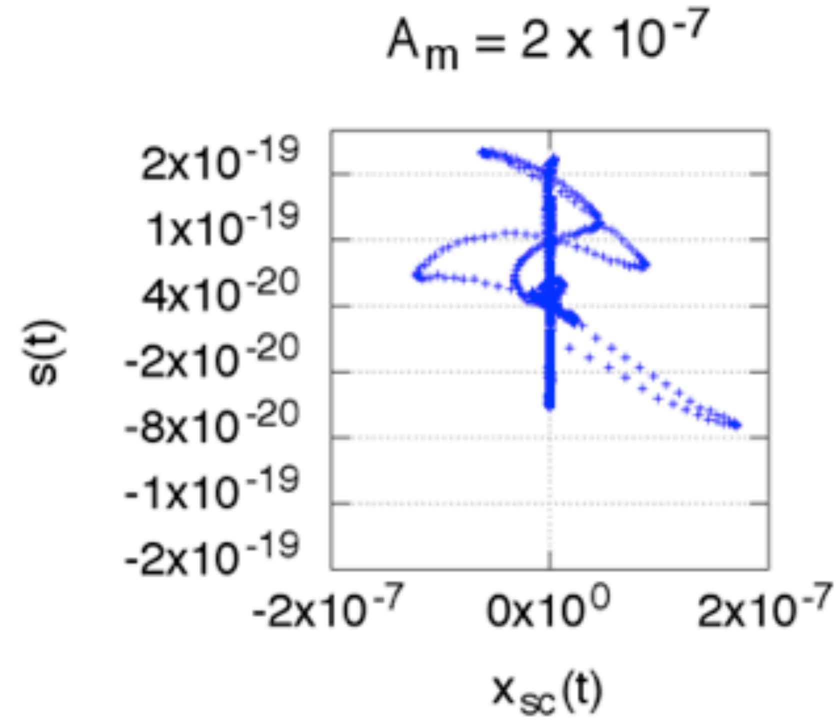
$\tau = 0.1[\mathrm{sec}]$: damping time
(estimated from Virgo paper)

$n_{seis}(t)$: stationary motion of mirror,
Assuming gaussian and stationary noise and S(f) = 10^{-8}[m/sqrtHz]

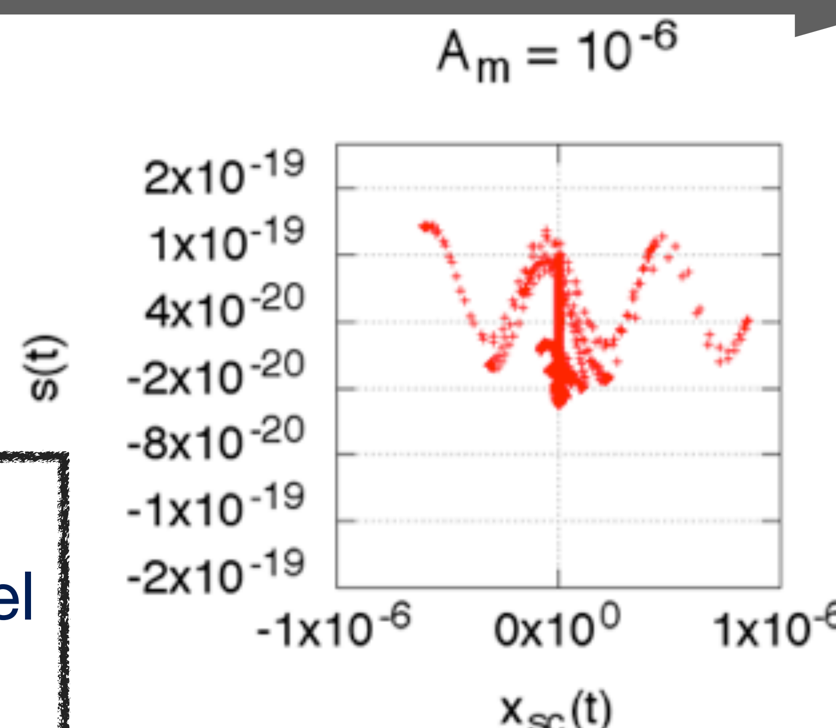
Scatter plot of simulated data

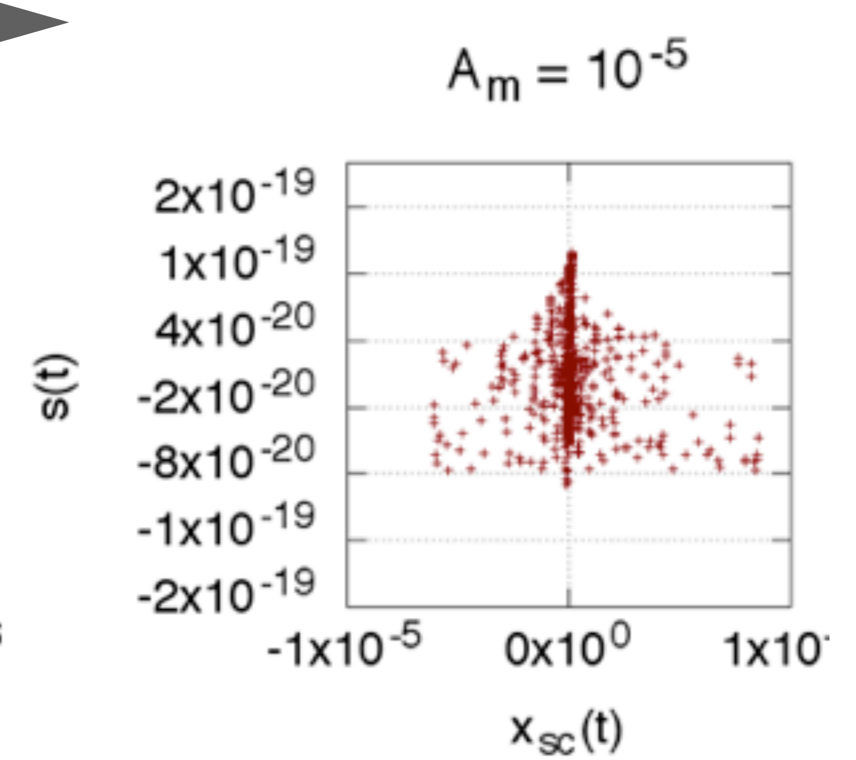






Weak Seismic Activity





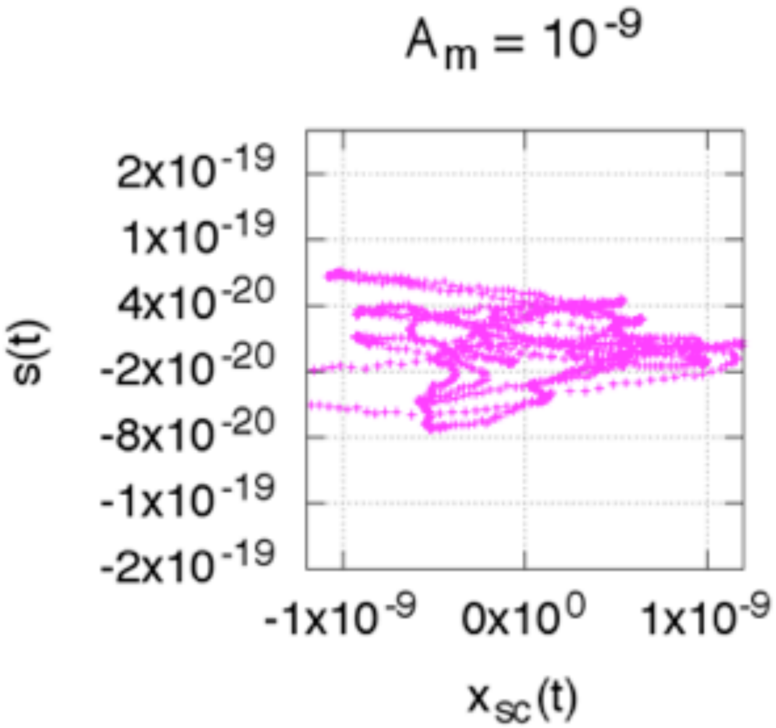
x-axis : displacement of mirror
y-axis : gravitational wave channel

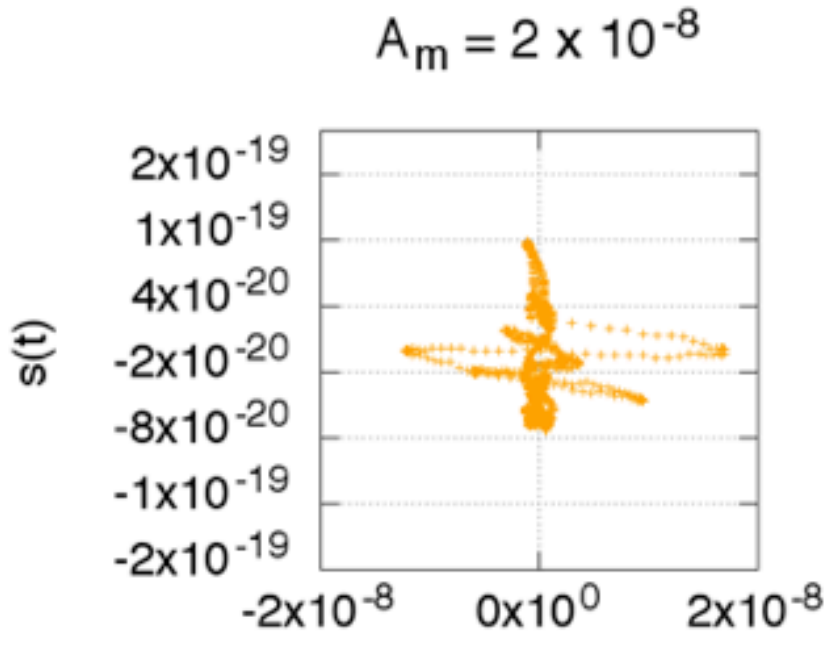
sampling rate = 1024[Hz]
duration = 1.0[s]

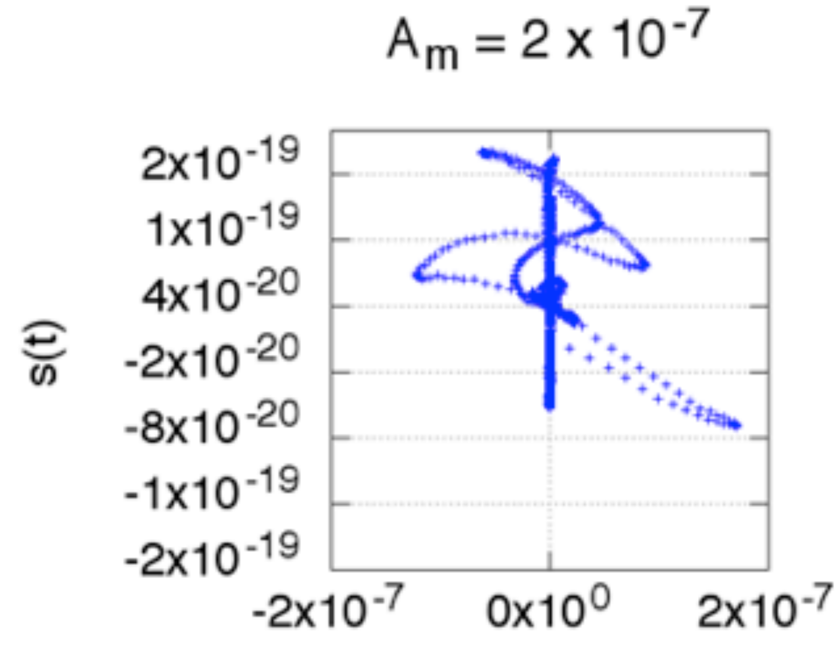
Strong Seismic Activity

Classification of relations between simulated data

no correlation | **linear correlation**

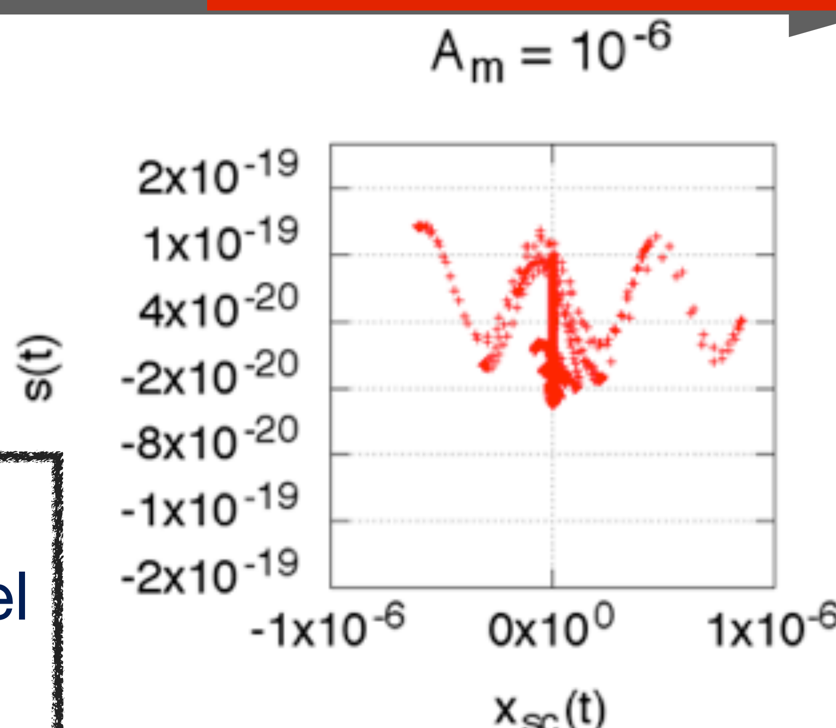


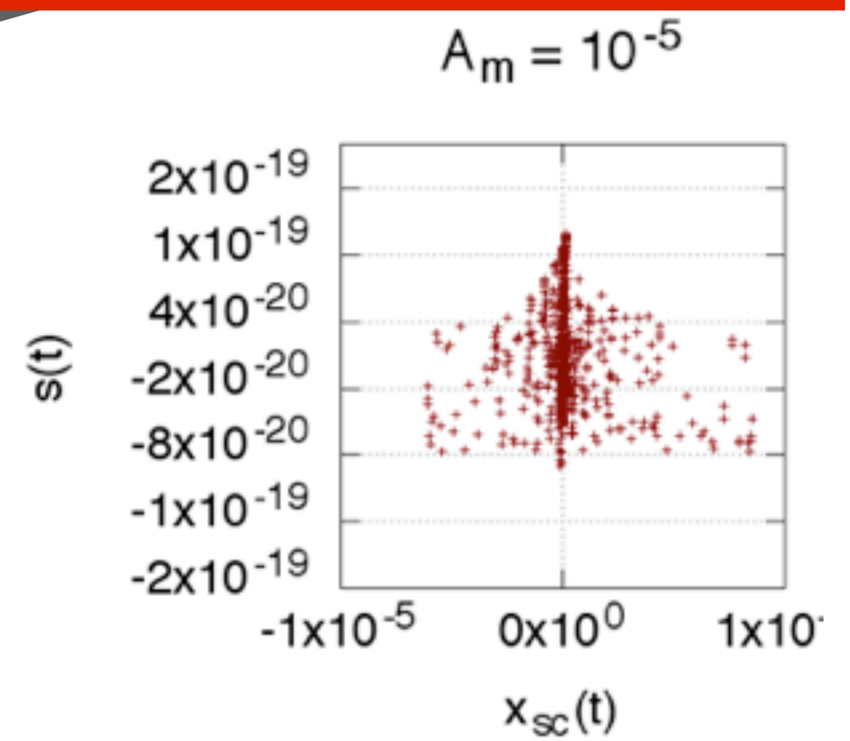




non-linear correlation (sine-shape)

Weak Seismic Activity





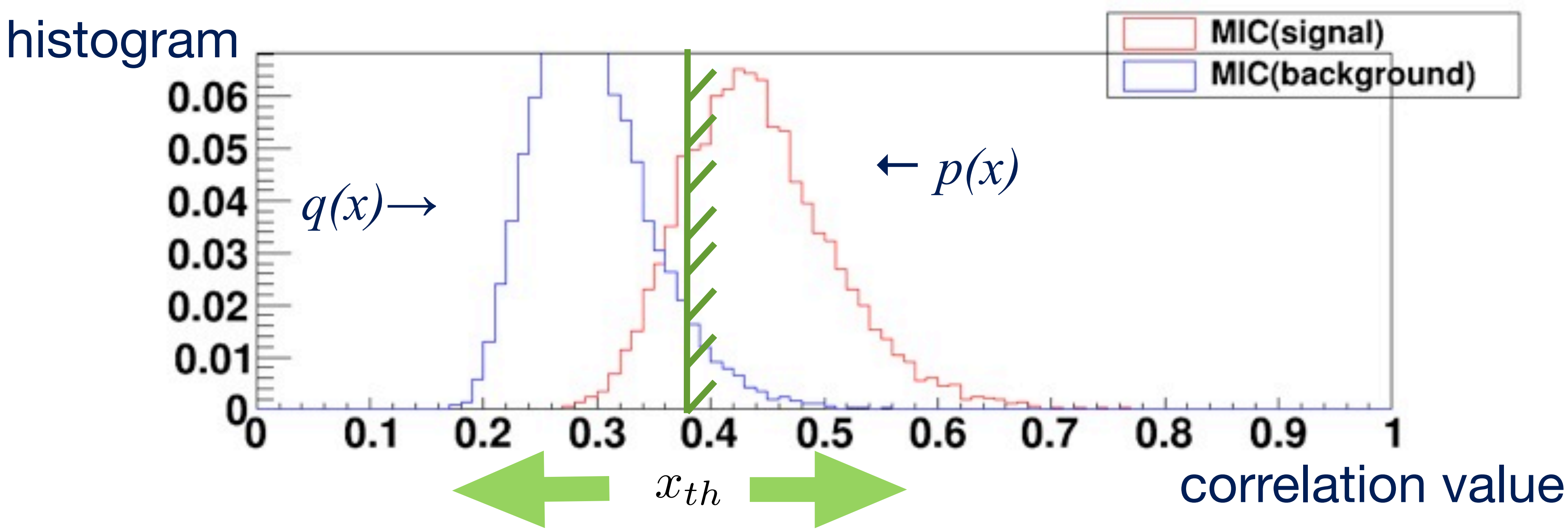
x-axis : displacement of mirror
y-axis : gravitational wave channel

sampling rate = 1024[Hz]
duration = 1.0[s]

Strong Seismic Activity

How to calculate ROC curve

We calculate Receiver Operating Characteristic(ROC) curve using obtained false alarm probability(FAP) and efficiency with 10000 trials.



For each histogram, we calculate false alarm probability(FAP) and detection efficiency at each threshold x_{th},

$$\text{false alarm probability}(x_{th}) = \sum_{x>x_{th}} p(x_i)/p_{total}$$

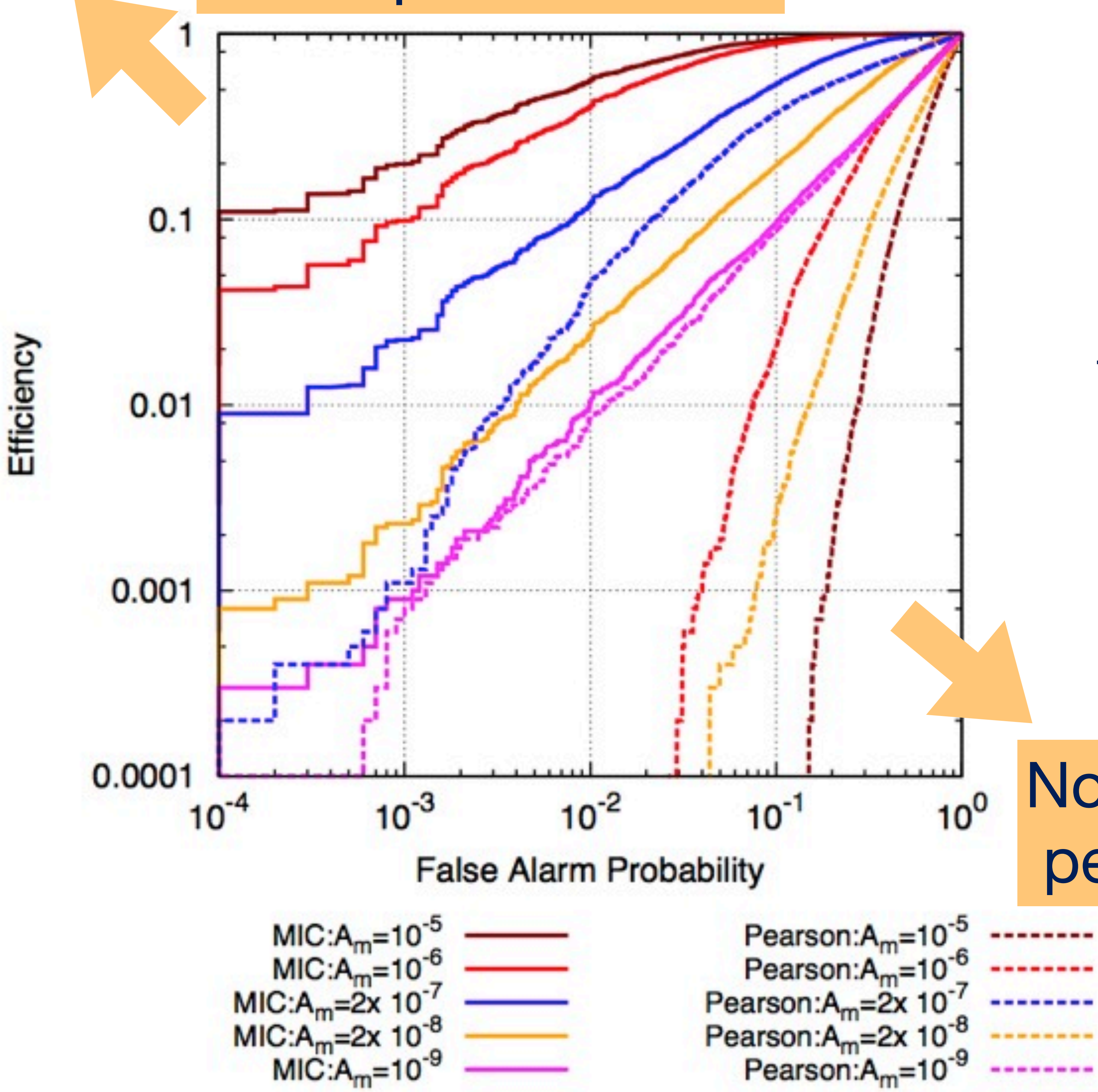
$$\text{efficiency}(x_{th}) = \sum_{x>x_{th}} q(x_i)/q_{total}$$

$$p_{total} = \sum_i p(x_i)$$

$$q_{total} = \sum_i q(x_i)$$

Evaluated performance of analysis methods - ROC curve

Good performance



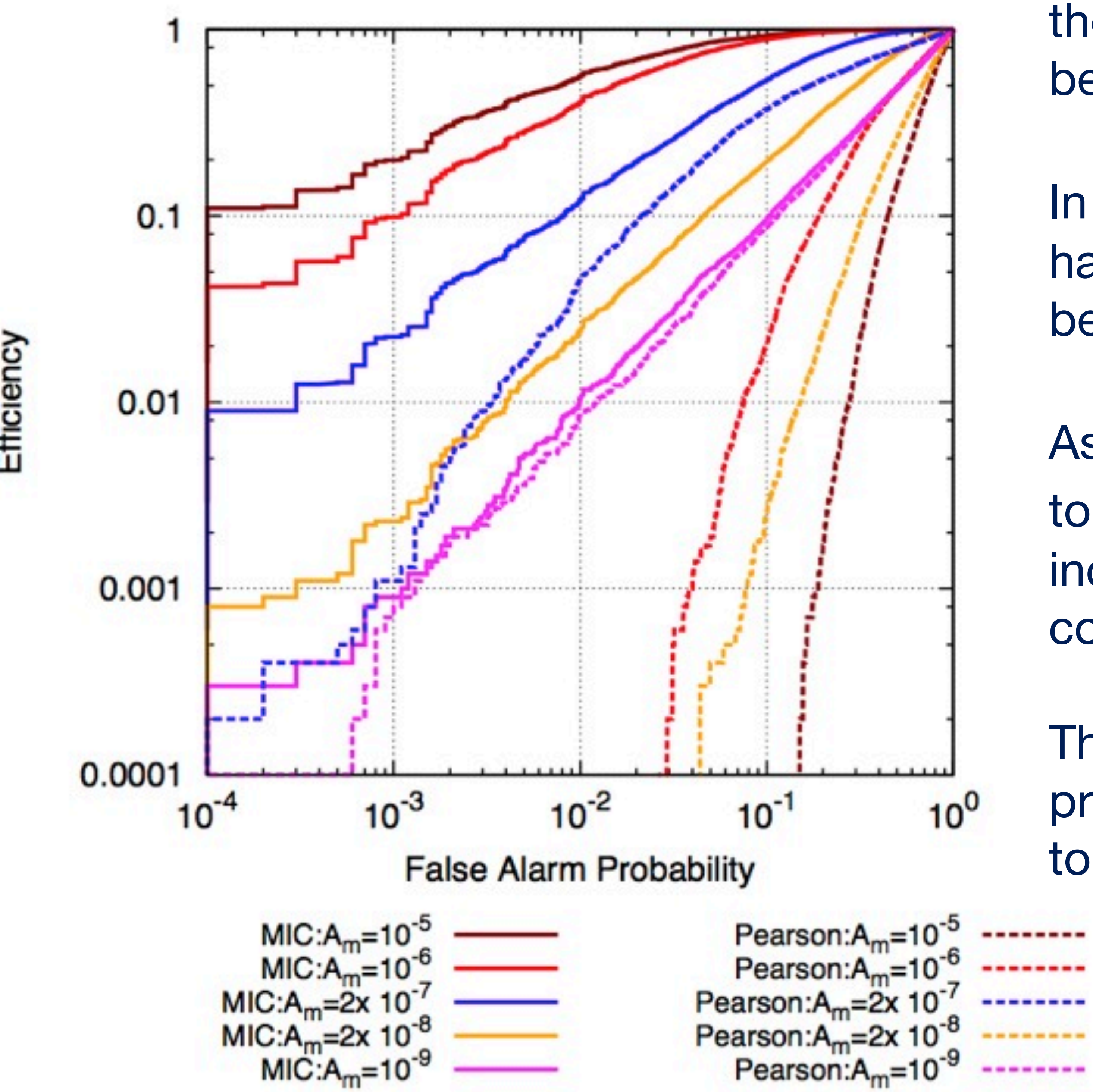
Not good performance

Calculated ROC curve indicates the performance of each method (MIC and Pearson).

If the ROC curve is close to the edge of top left, the method has good performance.

If the ROC curve is close to the edge of bottom right, the method has not good performance.

Evaluated performance of analysis methods - ROC curve



In the case of **Am = 10^{-9}** and **Am = $2x10^{-8}$**, the efficiency of Pearson and MIC is low, because data has no correlation.

In the case of **Am = $2x10^{-7}$**, the Pearson has a good performance, because data has linear correlation.

As noise origin increase from **Am = $2x10^{-7}$** to **Am = 10^{-5}**, the performance of MIC increase, because MIC can find linear correlation as well as non-linear correlation.

This result shows that the MIC is a promising method to find a non-linear relationship.

Summary

o In the past operations of GW detectors, the correlated noise between multiple channels participated in preventing the achievement of the design sensitivity. The unknown noise still remains.

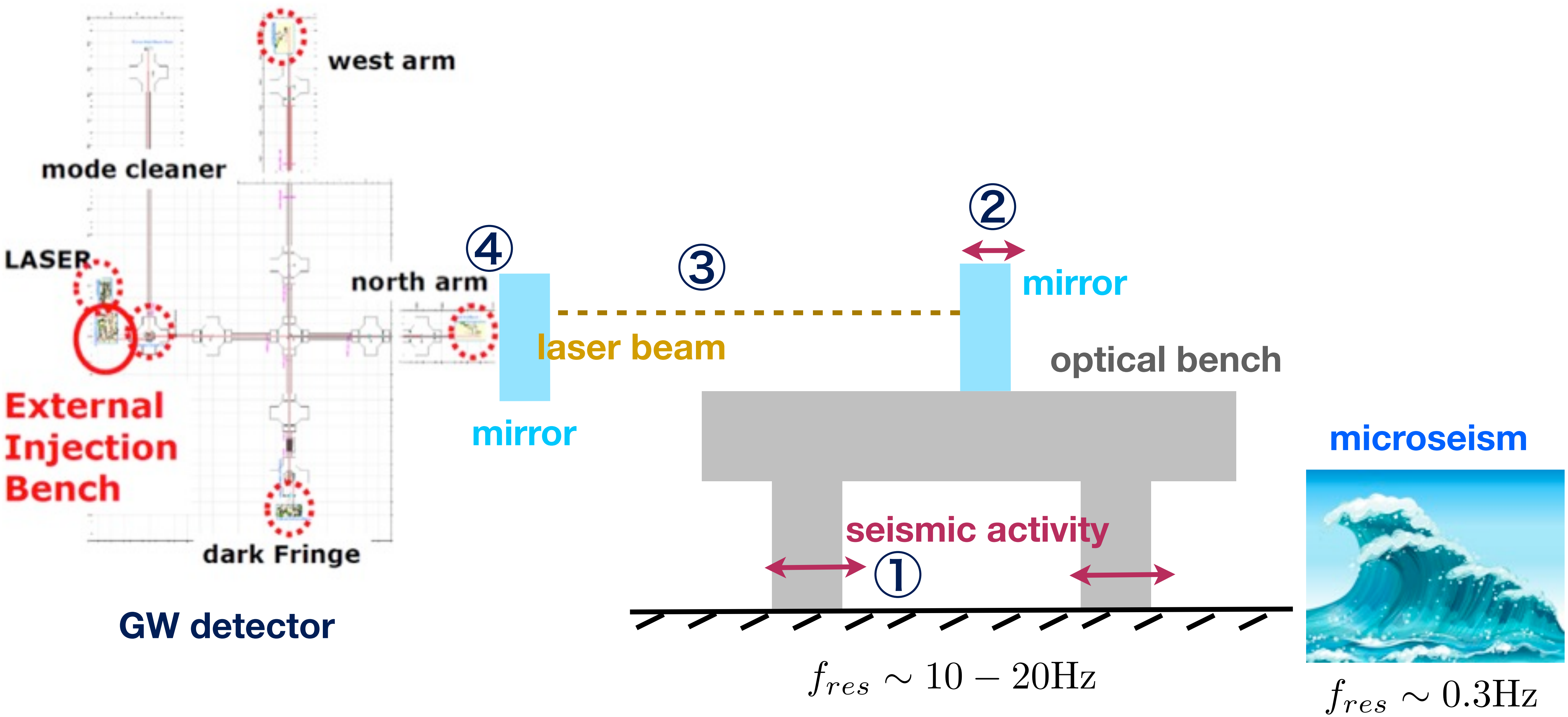
o We introduced the up-conversion noise and its model which is observed in Virgo detector and well-modeled.

o We propose the maximal information coefficient(MIC) to find non-linear correlation as well as linear correlation.

o The simulated ROC curve shows that the MIC can find the non-linearly correlated noise more efficiently than the Pearson correlation method.

Mechanism of this up-conversion

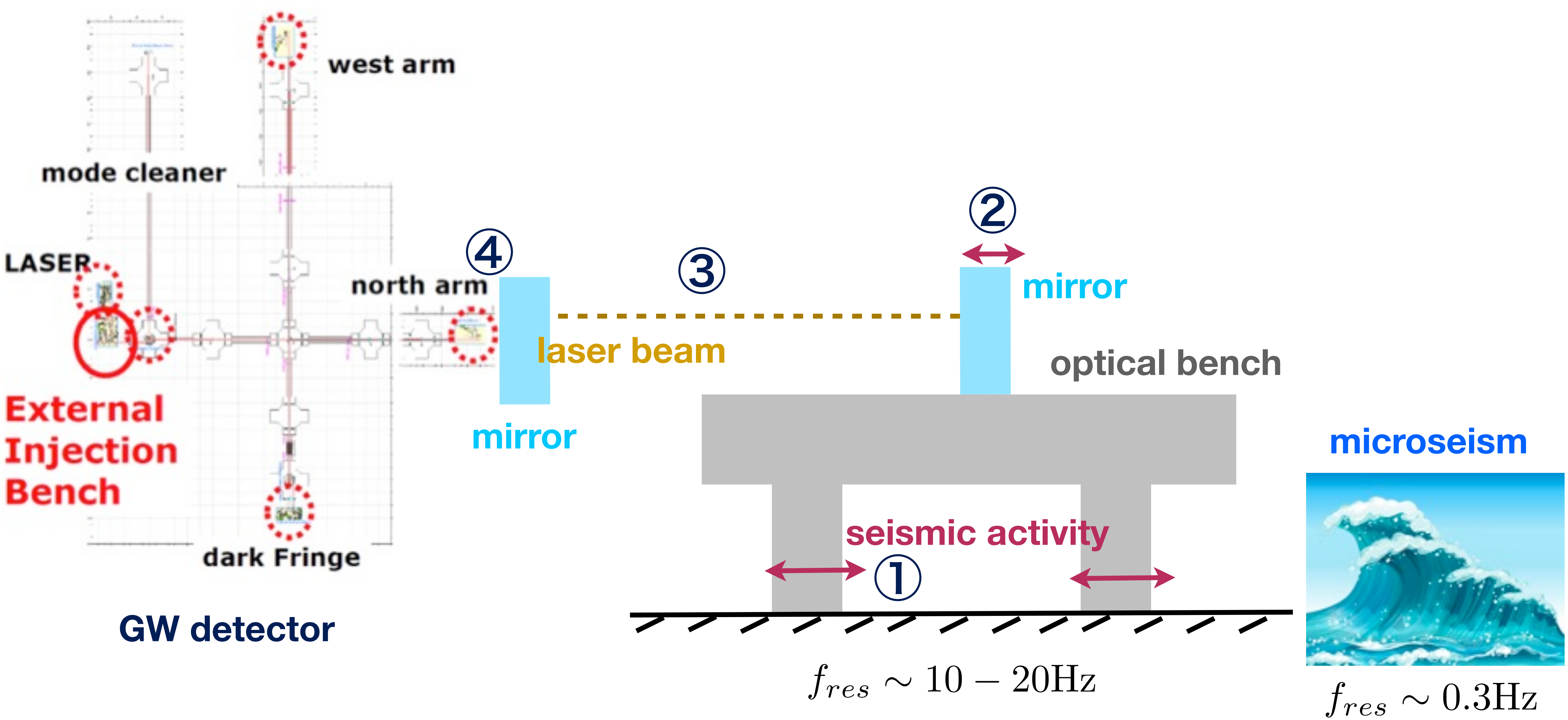
[Classical and Quantum Gravity 27, 19 (2010) 194011]



① Strong seismic activity (such as microseism..) excite resonant motion of optical bench and generate damping motion of optical bench.

Mechanism of this up-conversion

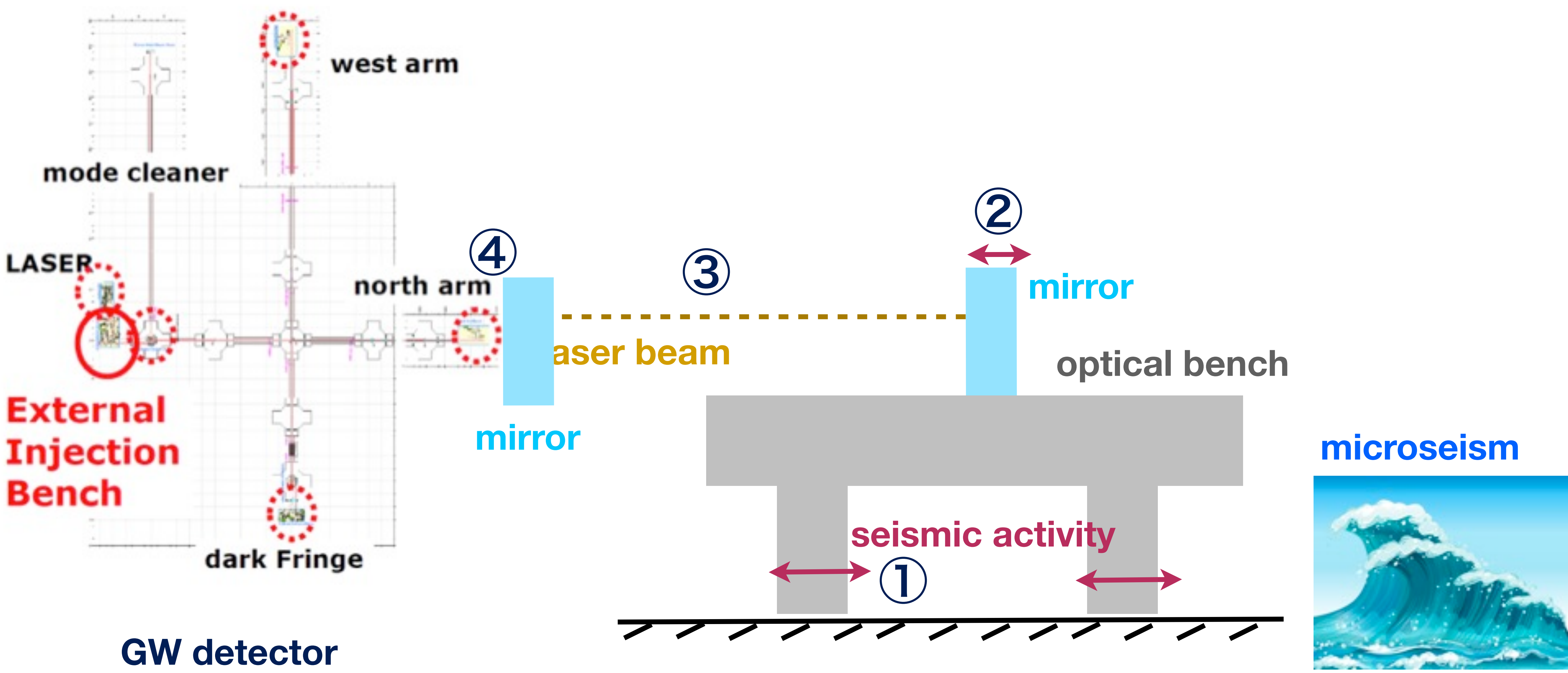
[Classical and Quantum Gravity 27, 19 (2010) 194011]



② The motion of optical bench causes damping motion of mirror installed on optical bench

Mechanism of this up-conversion

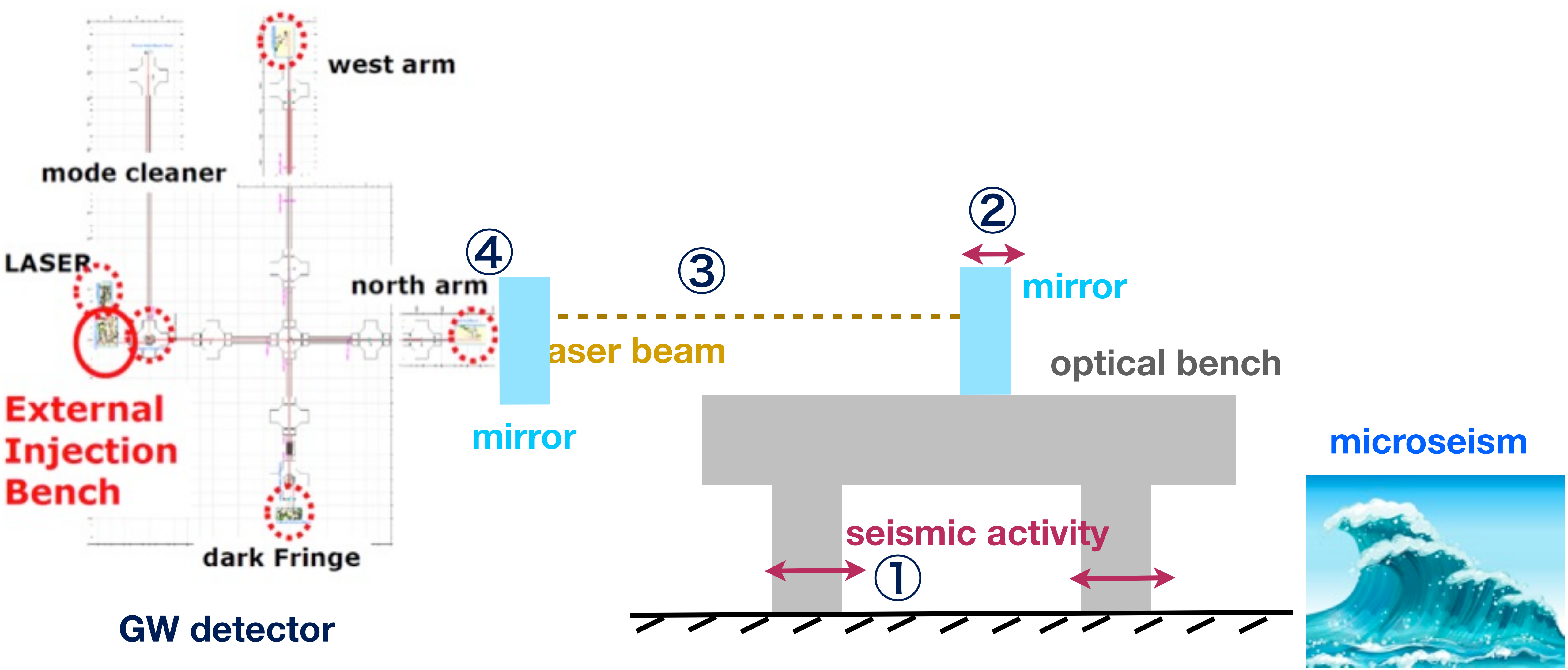
[Classical and Quantum Gravity 27, 19 (2010) 194011]



③ Time variation of optical path length between end-mirror of cavity and mirror on optical bench because of damping motion of mirror on optical bench

Mechanism of this up-conversion

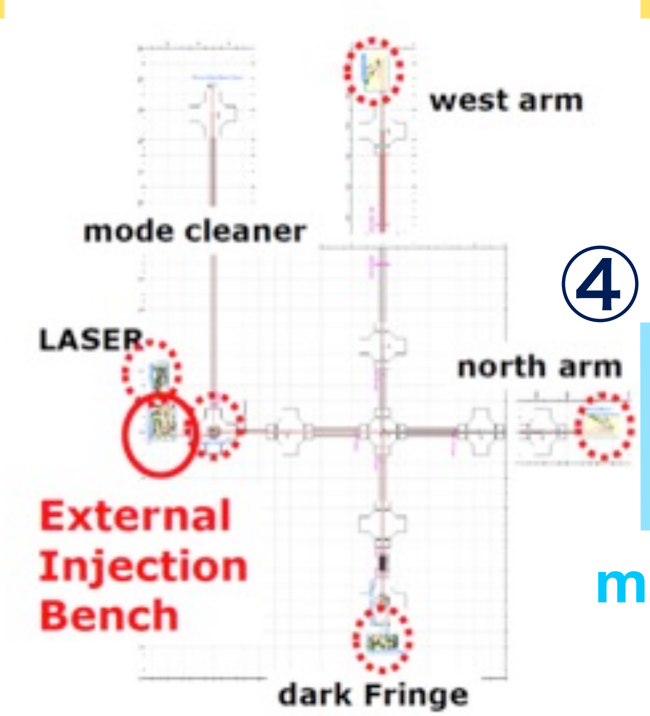
[Classical and Quantum Gravity 27, 19 (2010) 194011]

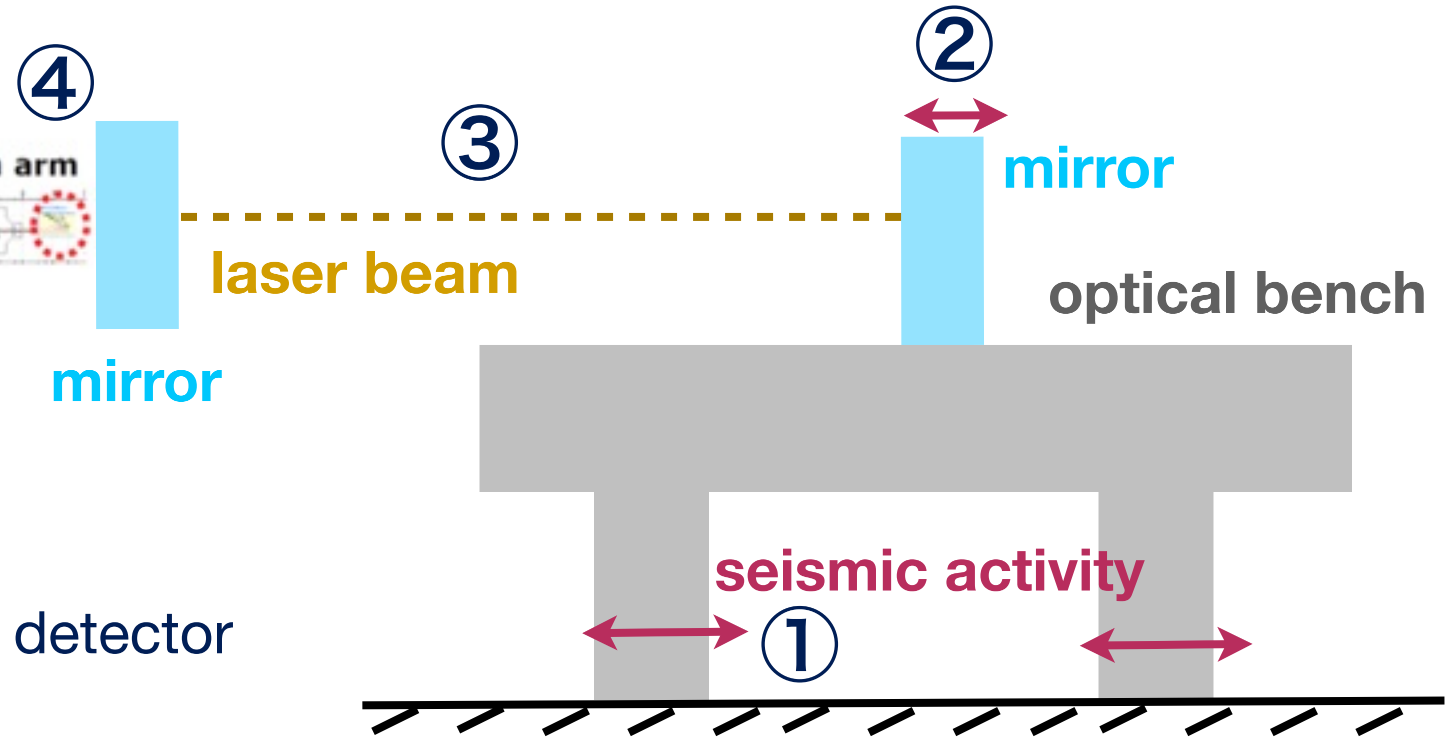


④ After modulated laser is returned to cavity, modulated laser will be noise source because of different phase.

Mechanism of this up-conversion

[Classical and Quantum Gravity 27, 19 (2010) 194011]





Optical system behind end-mirror controls GW detector using transmitted laser.
Sometimes accidentally transmitted laser is returned to cavity.

① Strong seismic activity (such as microseism) excite resonant motion of optical bench and generate damping motion of optical bench.
② damping motion of mirror installed on optical bench
③ time variation of optical path length between end-mirror and mirror on optical bench because of damping motion of mirror on optical bench
④ After modulated laser is returned to cavity, modulated laser will be noise source because of different phase.