A method for non-linear correlated noise in gravitational waves detectors

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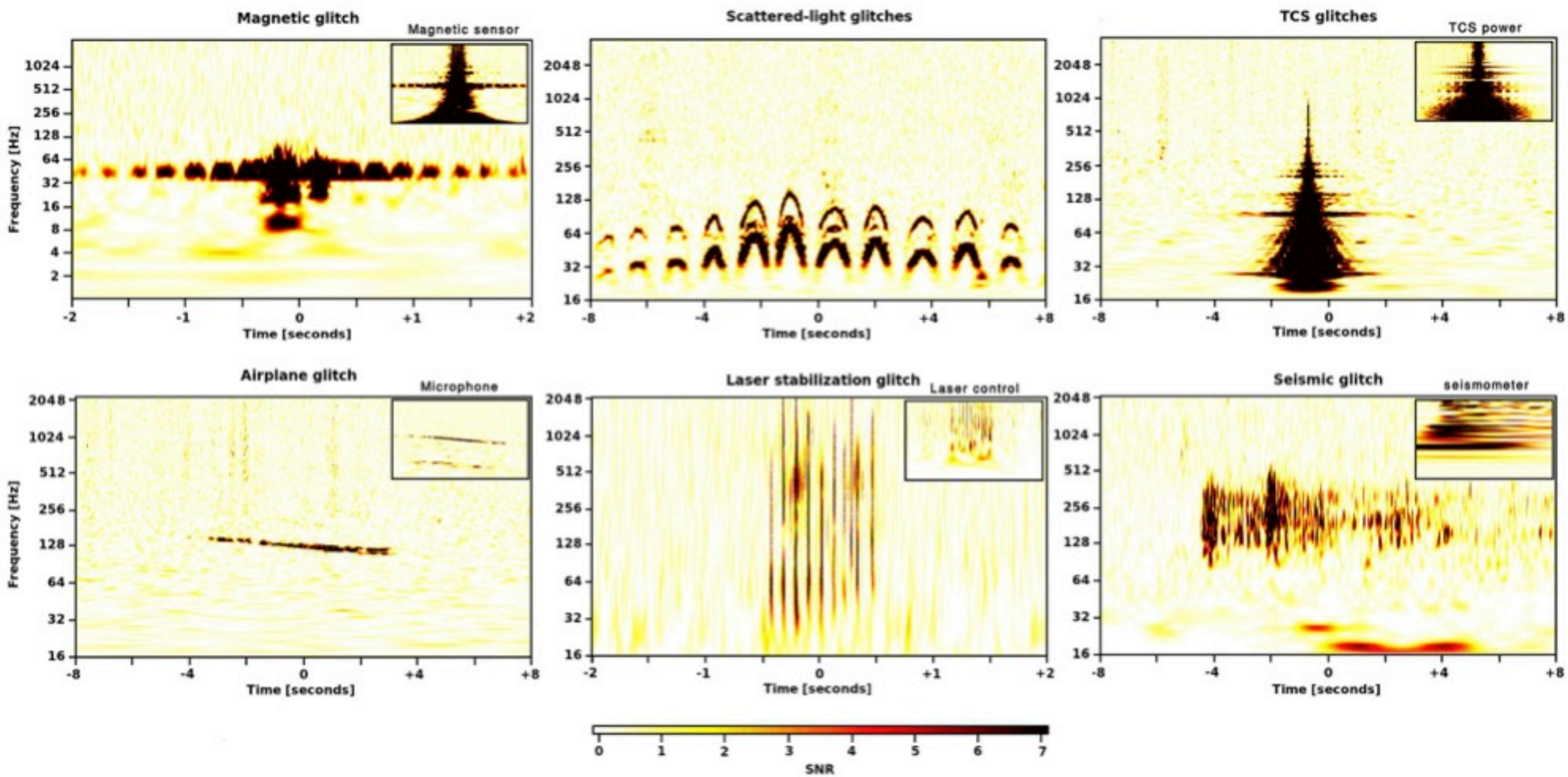
Example of the correlated noise observed in LIGO and Virgo

[J.Asis et al. Class.Quant.Grav. 29 (2012)]



These glitches are possible to behave as the gravitational wave signal.
We use the **correlation analysis** to find the correlated noise,
especially **non-linearly correlated noise**.

motivation - correlation analysis using PEM channels

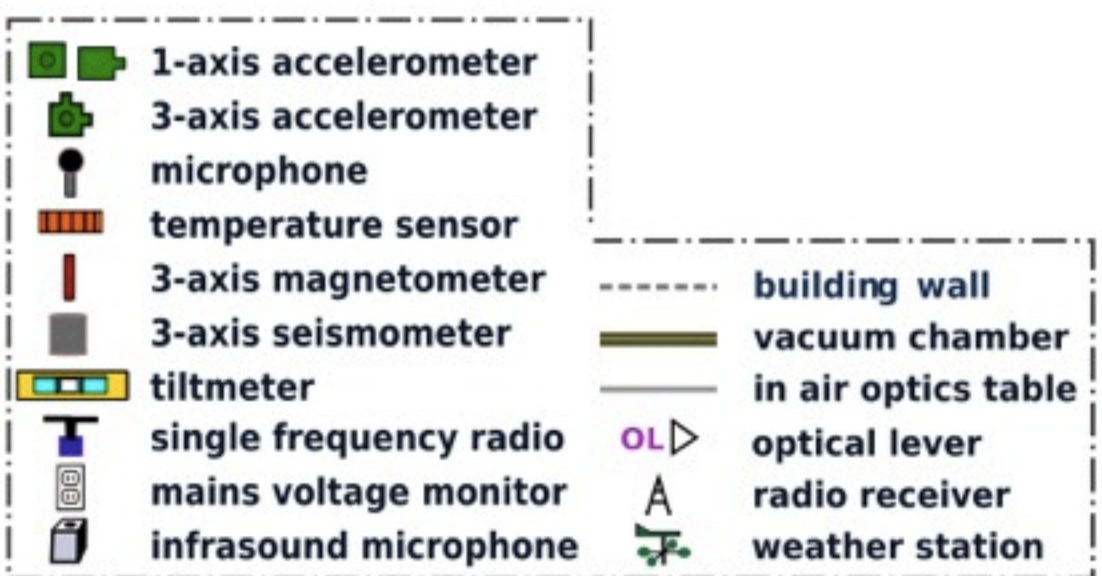
<u>Our Motivation : find correlated noise between ~10000 physical environmental channels and finally localize noise sources</u>

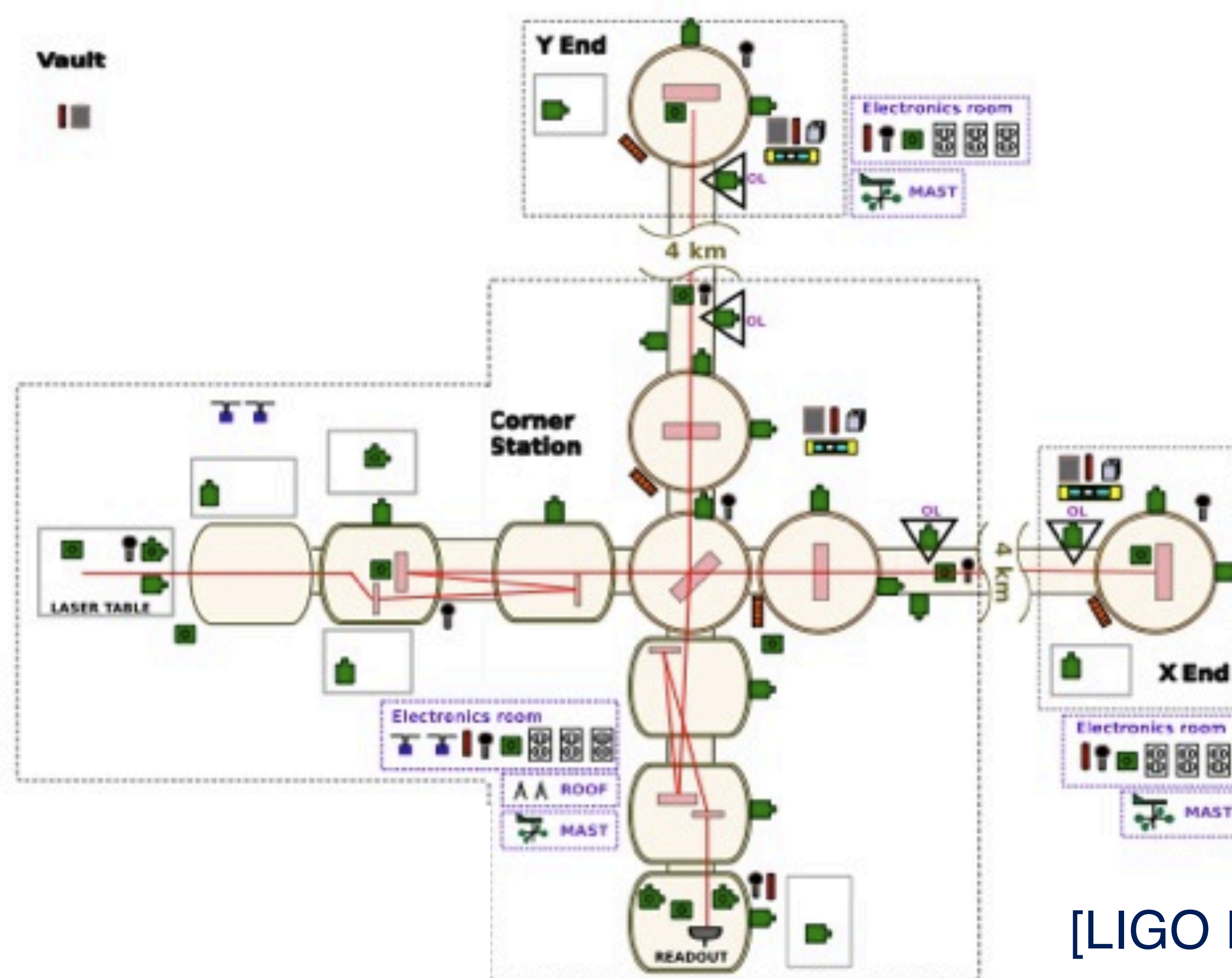
- Remove the noise sources to improve detector sensitivity
- Identify false trigger event generated by GW search pipeline

  -> the contribution to increase GW detection efficiency

In this talk,
- GW channel
  as sensitive channel to GW
- Physical envirnmental channel

such as,

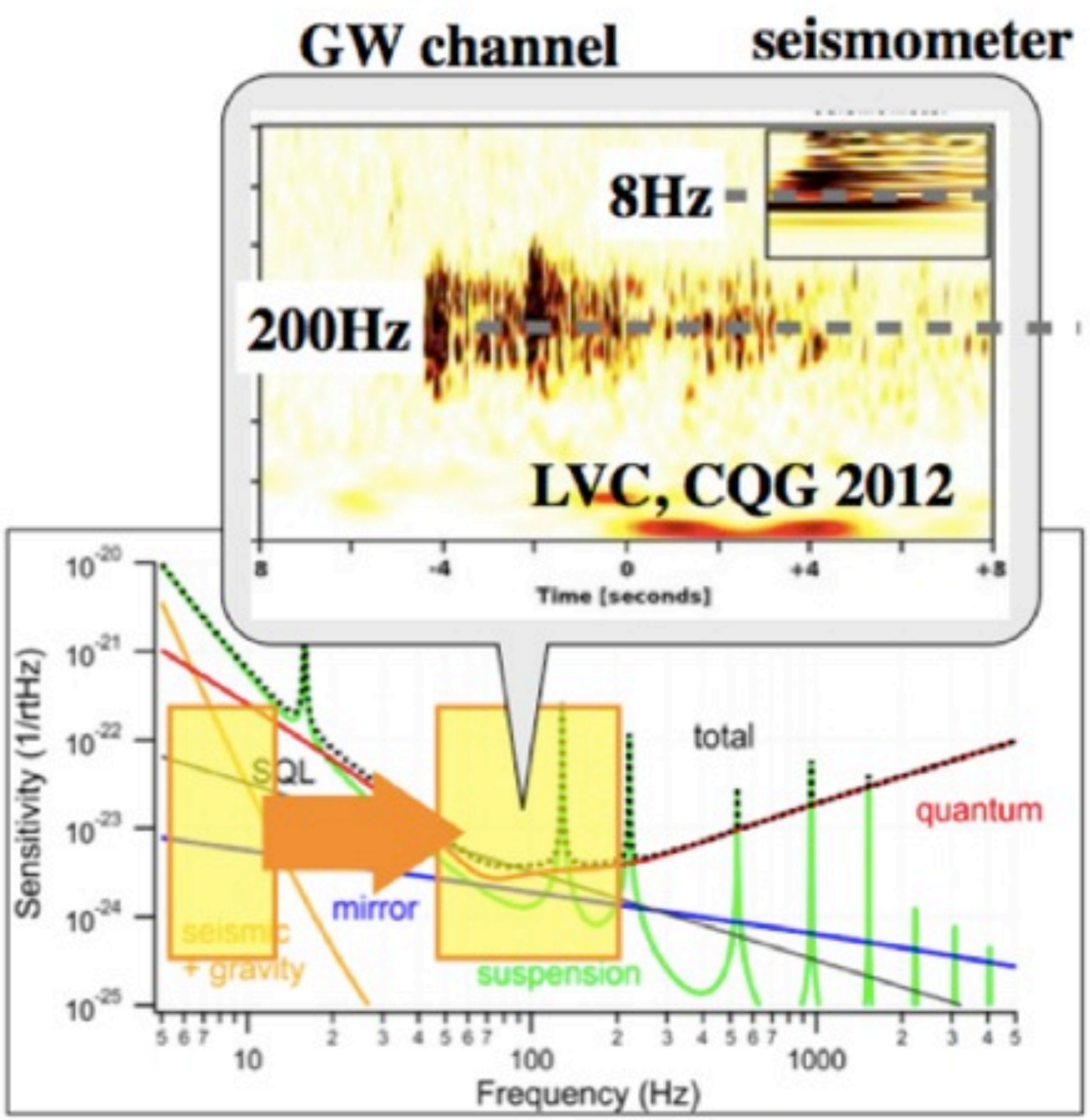




[LIGO P1500238]

~10000 physical environmental channels are online in LIGO Livingston.

Example of ***non-linear correlation*** *observed in LIGO and Virgo*



Sensitivity curve of KAGRA

J.Asis et al. Class.Quant.Grav. 29 (2012)
[CQG 27, 19 (2010) 194011]

Up-conversion noise
: one of the non-linearly correlated noise

low frequency $0.1 < f < 10Hz$,
such as seismic noise

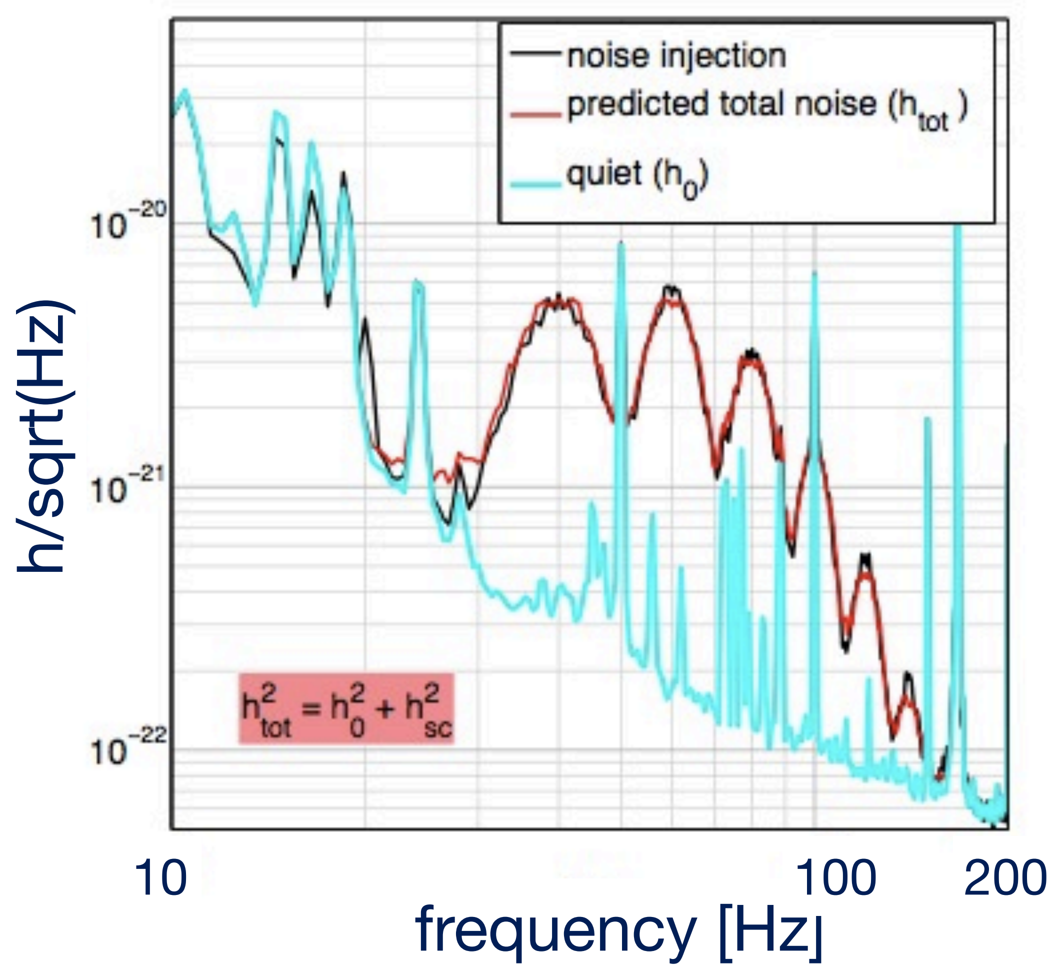
up-convert
(mechanism depends on the each noise and environment)

high frequency $10 < f < 1kHz$
such as scattered light noise in strain signal

The conventional correlation analysis methods can not detect a **non-linear correlation**.

Toward the near future, we should consider the analysis method for non-linearly correlation.

Up-conversion noise observed at Virgo detector



[Classical and Quantum Gravity 27, 19 (2010) 194011]

The past Virgo detector has been limited by this up-conversion noise.
The up-conversion noise has been solved and also **well-modeled**.

mechanism

1. Strong seismic activity force the oscillation of the optical bench which is installed behind the end mirror to control the detector.

2. The scattered light is appeared and modulated by the motion of the optical components.

3. This scattered light recombines with the main optical beam and appears as a phase noise in the GW strain signal.

Up-conversion noise model observed at Virgo detector

GW channel

[Classical and Quantum Gravity 27, 19 (2010) 194011]

$$s(t) = h_{sc}(t) + n(t)$$

n(t) : fundamental noise of GW detector
(Virgo sensitivity is used. Assuming gaussian and stationary noise)

up-conversion noise (effect of scattered light noise)

$$h_{sc}(t) = G \cdot \sin\left(\frac{4\pi}{\lambda}(x_0 + \delta x_{sc(t)})\right)$$

$\delta x_{sc}(t)$: displacement of mirror by seismic activity

x_0 : distance between end mirror and reflector

G : constant factor depending on interferometer $(G = 5 \times 10^{-20})$
λ : laser wavelength (1064 [nm])

displacement of mirror excited by seismic activity

$$\delta x_{sc}(t) = A_m \sin(2\pi f_m t) \exp(-t/\tau) + n_{seis}(t)$$

A_m : amplitude of mirror's displacement

$f_m = 15[\mathrm{Hz}]$: resonant frequency of optical bench

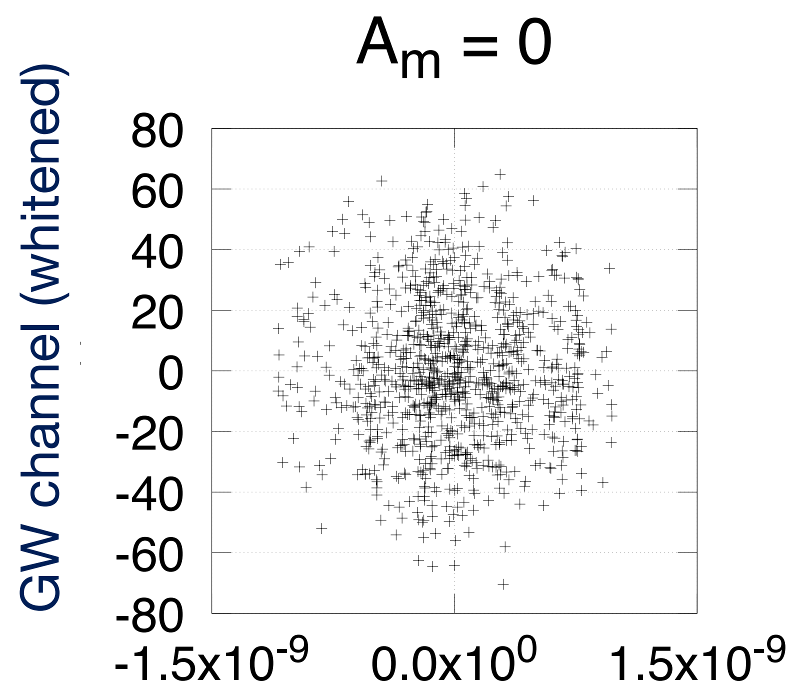
$\tau = 0.1[\mathrm{sec}]$: damping time
(estimated from Virgo paper)

$n_{seis}(t)$: stationary motion of mirror,
Assuming gaussian and stationary noise and S(f) = 10^{-8}[m/sqrtHz]

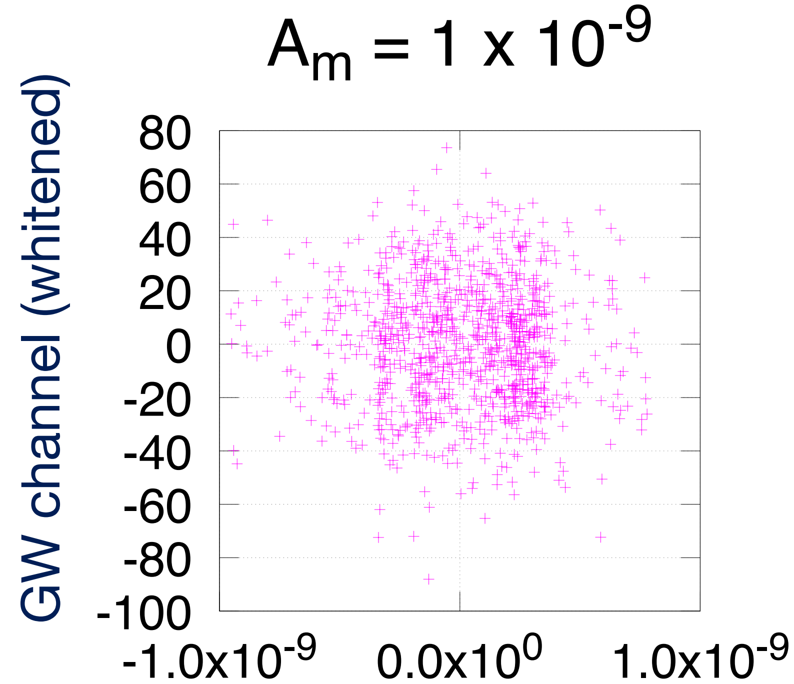
Following this noise model, we generate a set of simulated data.

Scatter plot of simulated data

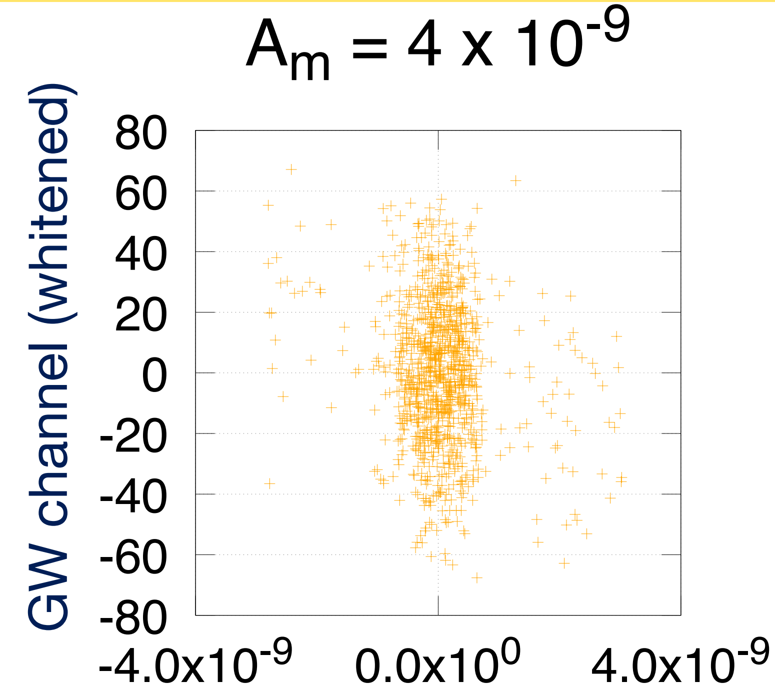
sampling rate = 1[kHz]
duration = 1.0[s]



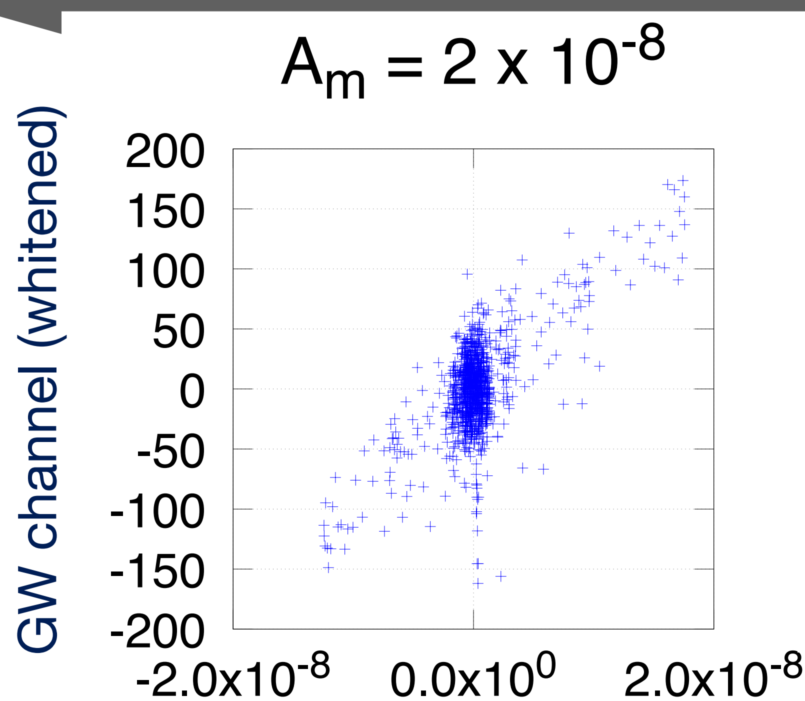
Weak Seismic Activity



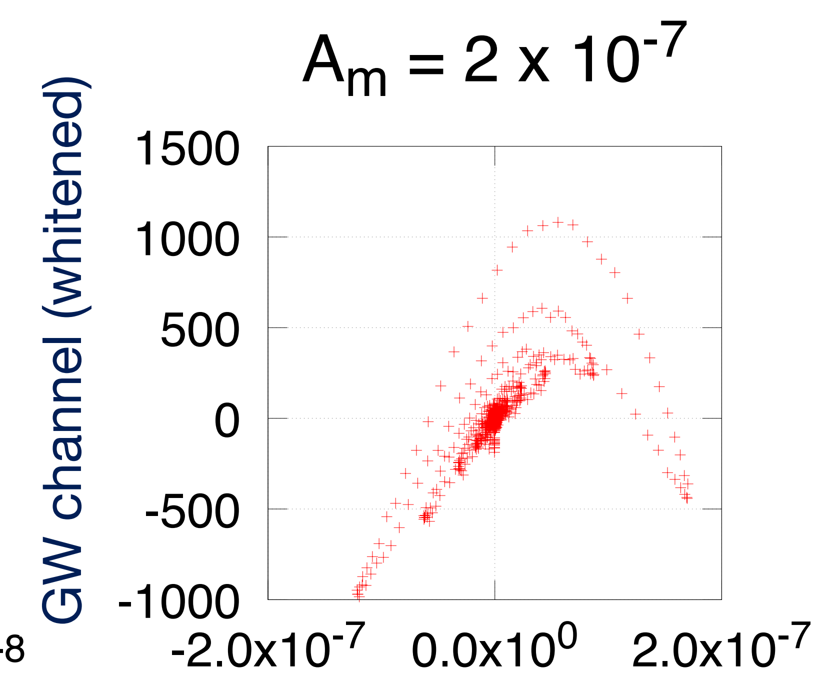
displacement of mirror



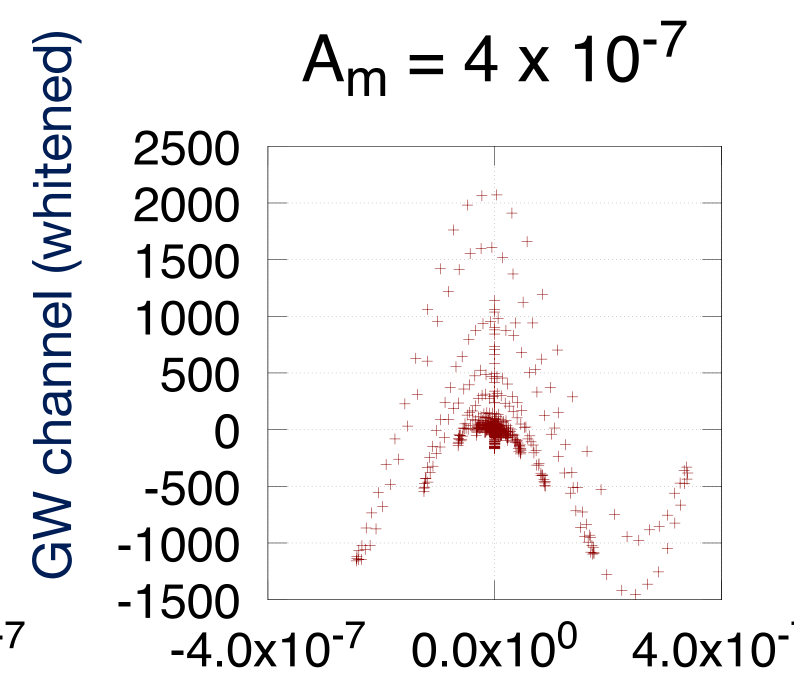
displacement of mirror



displacement of mirror



displacement of mirror



Strong Seismic Activity

Up-conversion noise model observed at Virgo detector

[Classical and Quantum Gravity 27, 19 (2010) 194011]

GW channel

$$s(t) = h_{sc}(t) + n(t)$$

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up-conversion noise (effect of scattered light noise)

$$h_{sc}(t) = G \cdot \sin\left(\frac{4\pi}{\lambda}(x_0 + \delta x_{sc(t)})\right)$$

$\delta x_{sc}(t)$: displacement of mirror by seismic activity

x_0 : distance between end mirror and reflector

G : constant factor depending on interferometer $(G = 5 \times 10^{-20})$

λ : laser wavelength (1064 [nm])

Such a change of relationship is caused by this term;

In the case of $\delta x_{sc}(t) \ll \frac{\lambda}{4\pi} \simeq 10^{-7}$, approximately $h_{sc}(t) \propto \delta x_{sc}(t)$.

=> linear correlation

In the case of $\delta x_{sc}(t) \geq \frac{\lambda}{4\pi} \simeq 10^{-7}$, $h_{sc}(t) = G \cdot \sin\left(\frac{4\pi}{\lambda}(x_0 + \delta x_{sc(t)})\right)$

=> non-linear correlation

Correlation analysis methods

In this study, we consider two methods;

- Pearson Correlation Coefficient
  (efficient method to linear correlation)

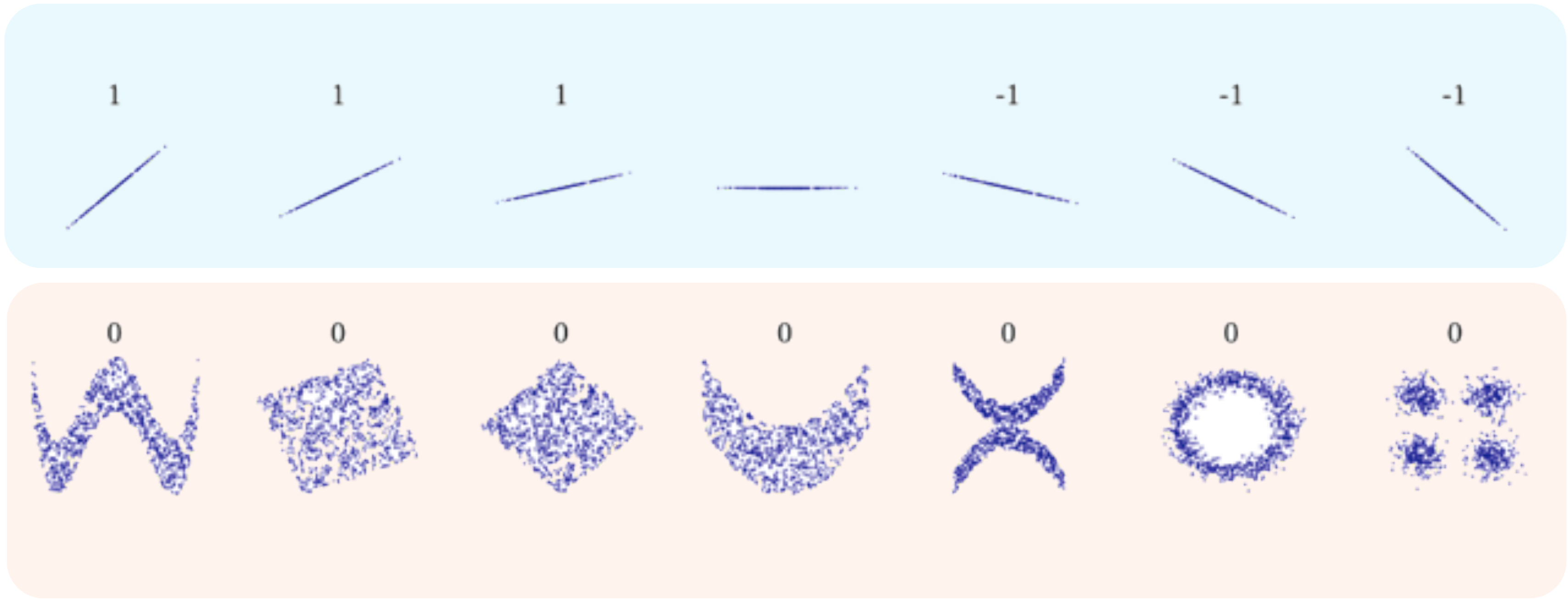
$$r = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}}$$

- **Maximum Information Coefficient (MIC)**
  (MIC can find a non-linear correlation as well as linear correlation.)

[David N. Reshef, et al. Science 334, 1518 (2011)]

Linear correlation →

non-linear correlation →



http://lectures.molgen.mpg.de/algsysbio12/MINEPresentation.pdf

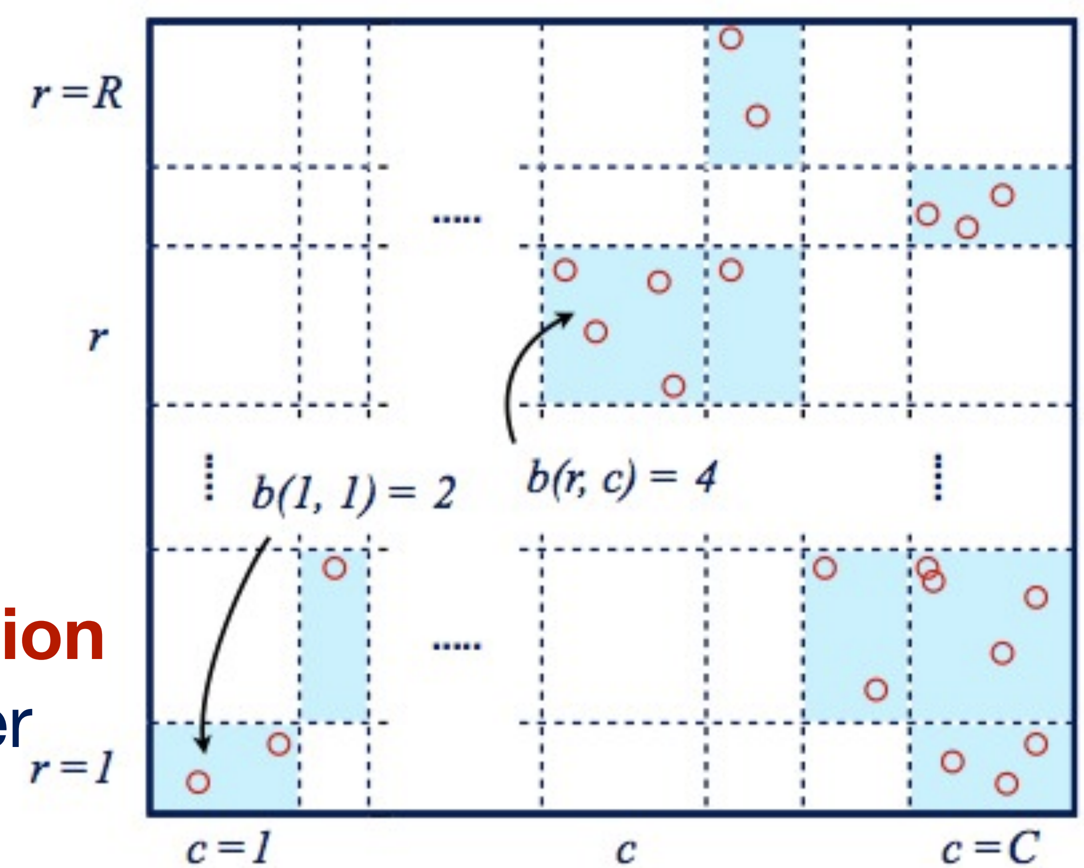
Maximum Information Coefficient (MIC)

[David N. Reshef, et al. Science 334, 1518 (2011)]

o If a relationship exists between two channels, grids can be drawn on the scatter plot of two data that partitions the data to catch that relationship.

(1) For each placement of partition, the mutual information is calculated.

$$I(R,C) = \sum_{r=1}^{R}\sum_{c=1}^{C} p(r,c)\log_2\frac{p(r,c)}{p(r)p(c)},$$



p(r, c) : joint probability mass function
p(r), p(c) : marginal probability function

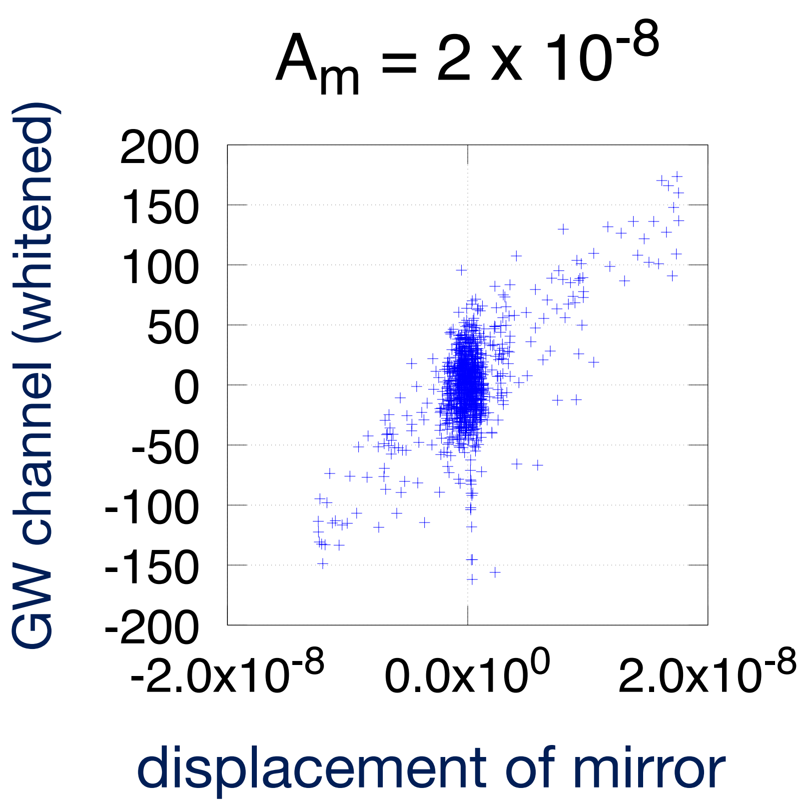
(2) The MIC is defined as **the mutual information maximized under all the possible grids** under RC < B(N). B(N) is maximal number of cell and we use $B(N) = N^{0.6}$ now.

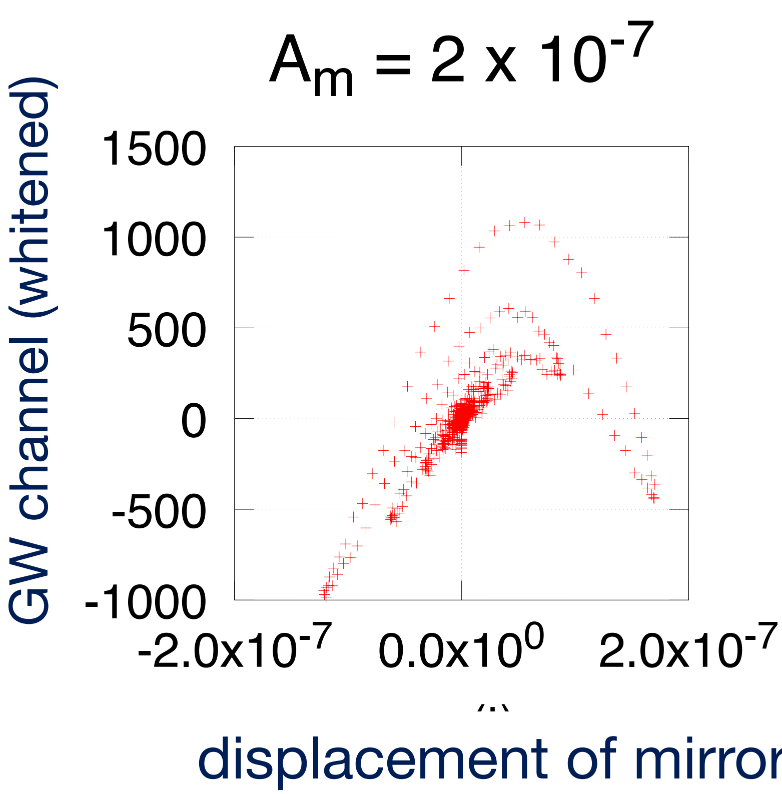
$$MIC(x,y) = \max_{RC<B(N)} \frac{I(R,C)}{\log_2(\min\{R,C\})},$$

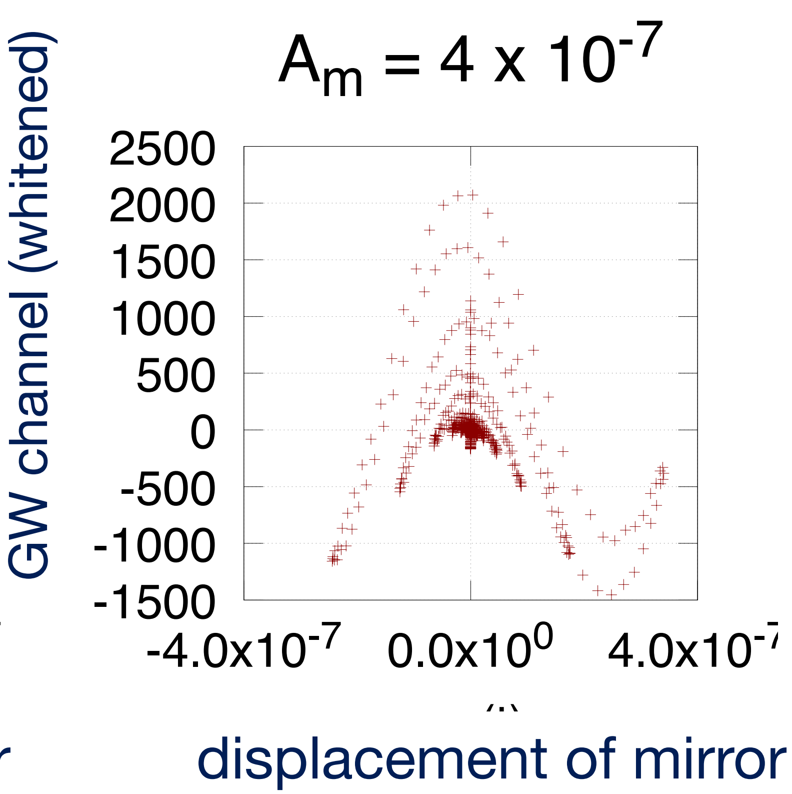
Estimate performance of methods

In this study, following the model of the up-conversion noise, we generate a set of simulated data;
the displacement of mirror and whitened GW channel,

We apply Pearson and MIC to the simulated data.
Using the estimated Receiver Operating Characteristic(ROC) curve, we show the performance of methods to the linear and non-linear correlation.

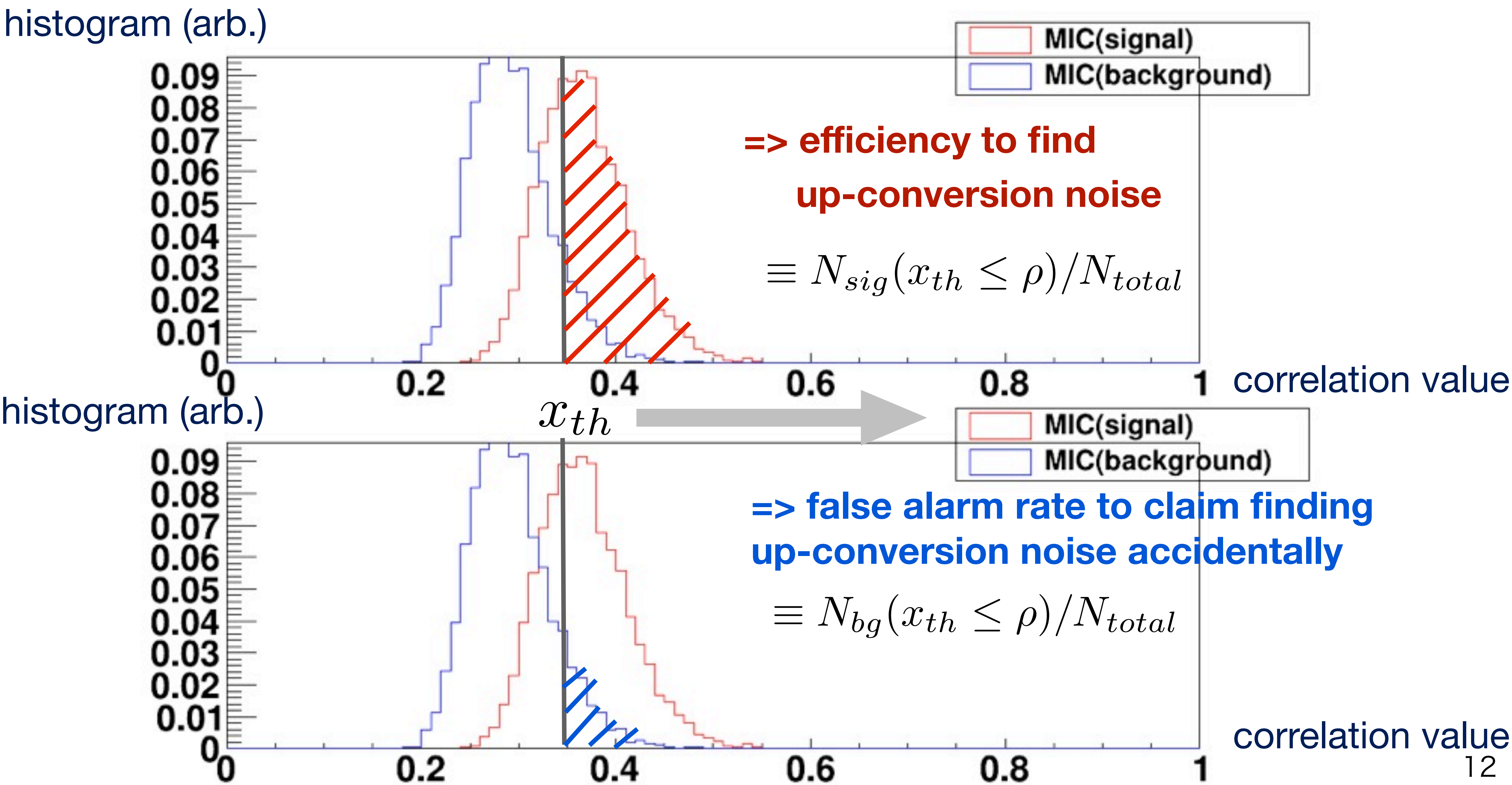






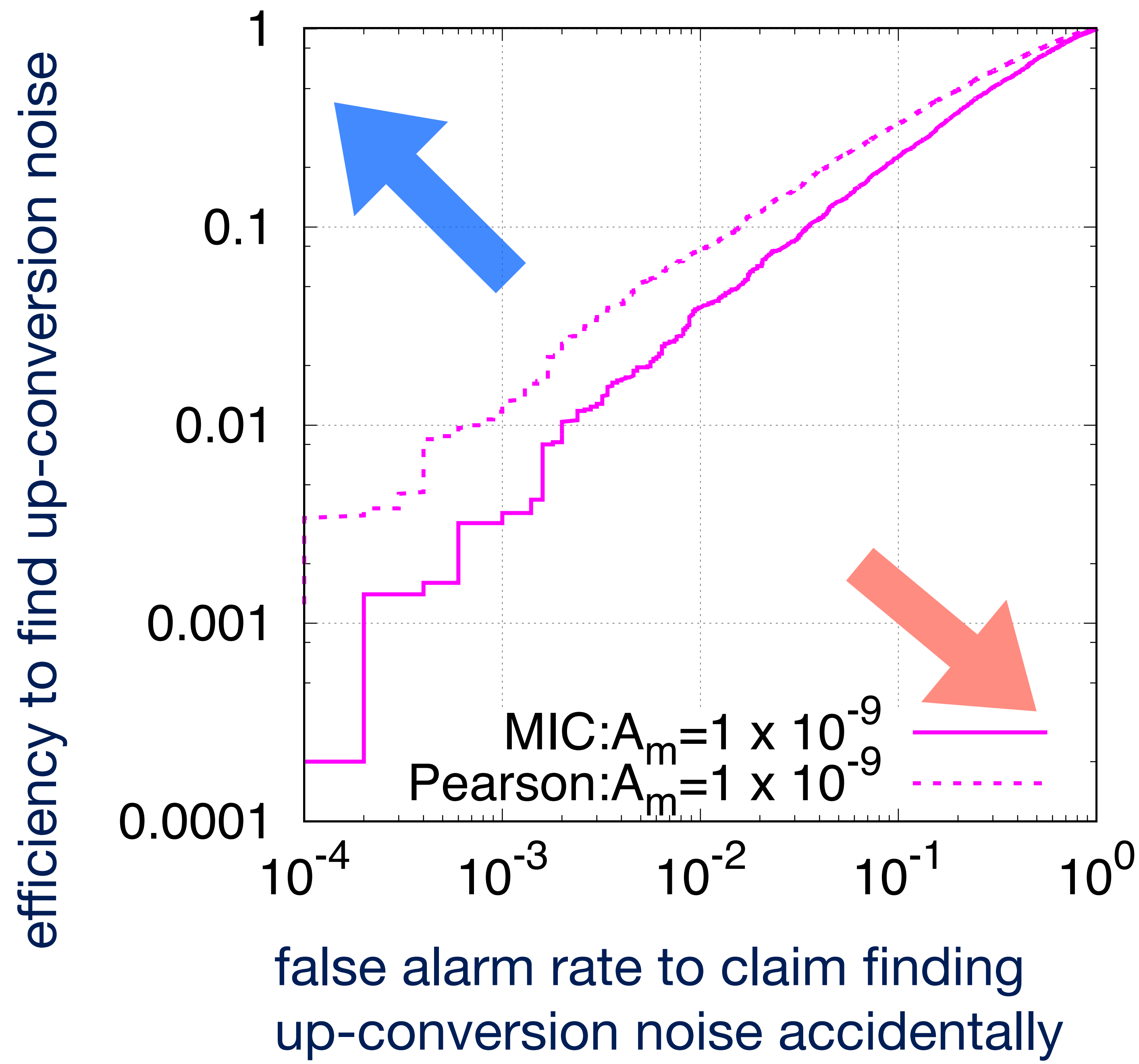
How to calculate ROC curve

- o Monte Carlo simulation 10000trials => Receiver Operating Characteristic(ROC)
- o **red histogram** : the simulation with up-conversion noise (e.g. $A_m = 4 \times 10^{-9}$)
  blue histogram : the simulation without up-conversion noise $A_m = 0.0$



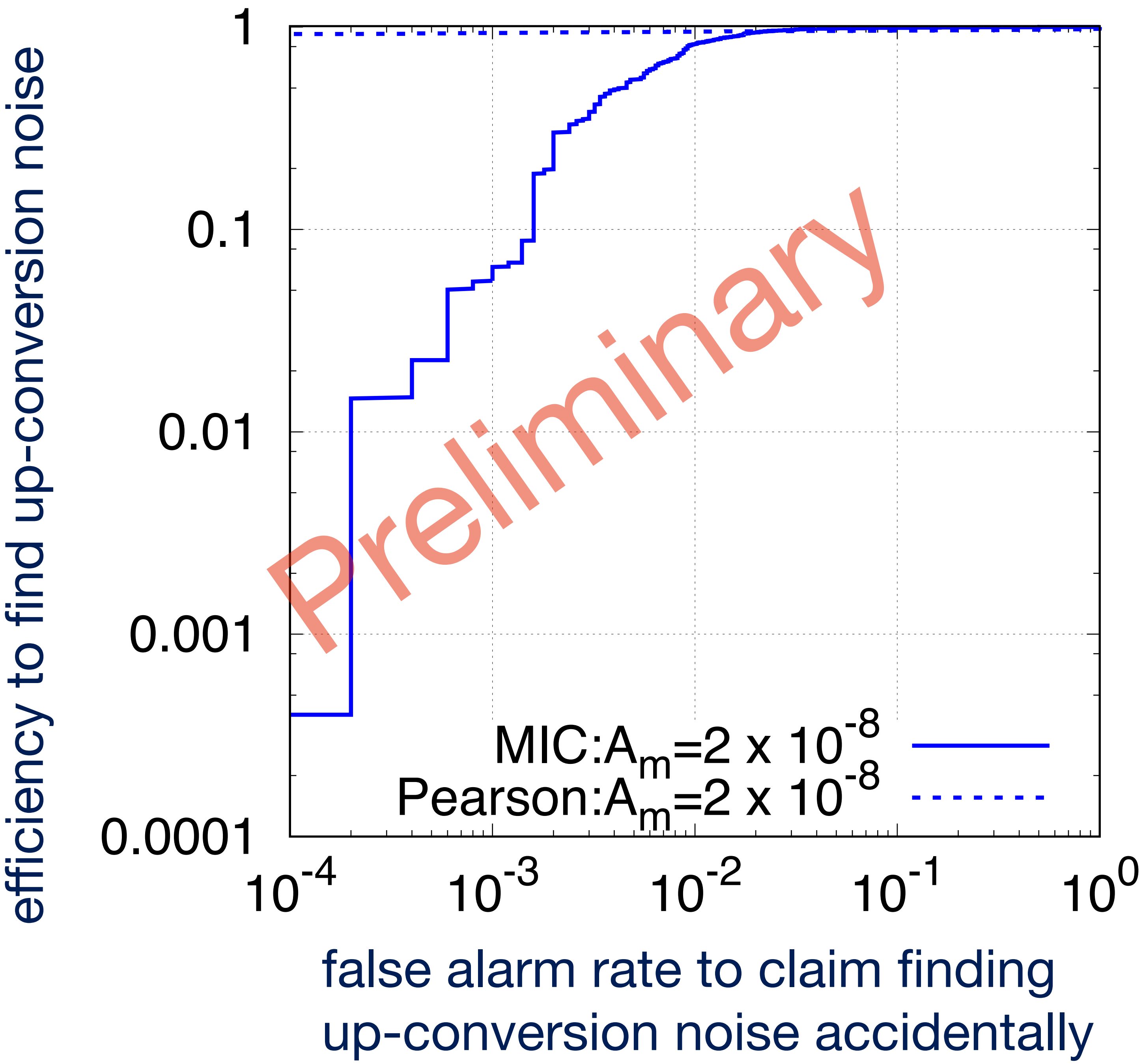
Receiver Operating Characteristic(ROC) curve

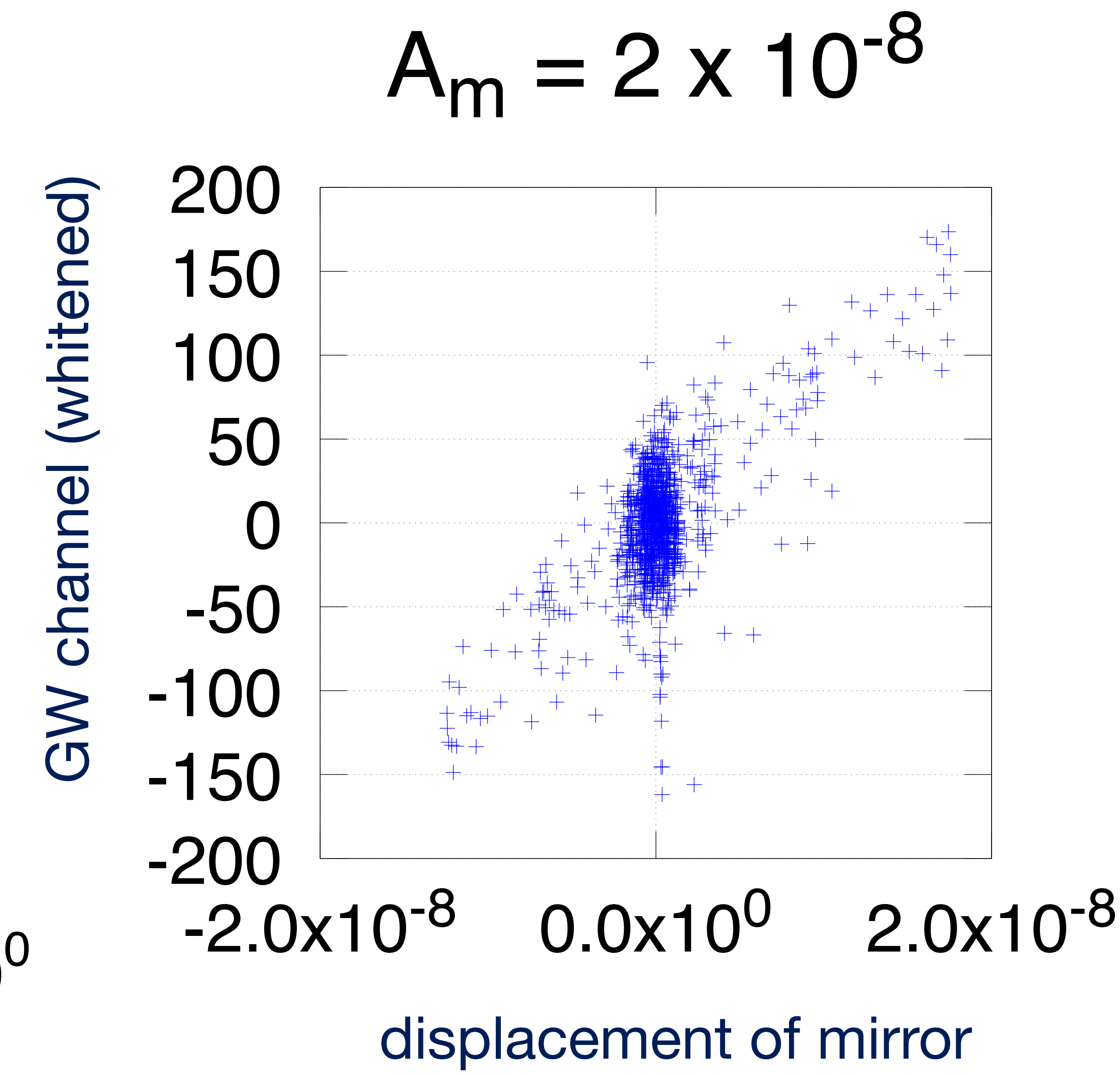
Better performance to find up-conversion noise



Worse performance to find up-conversion noise

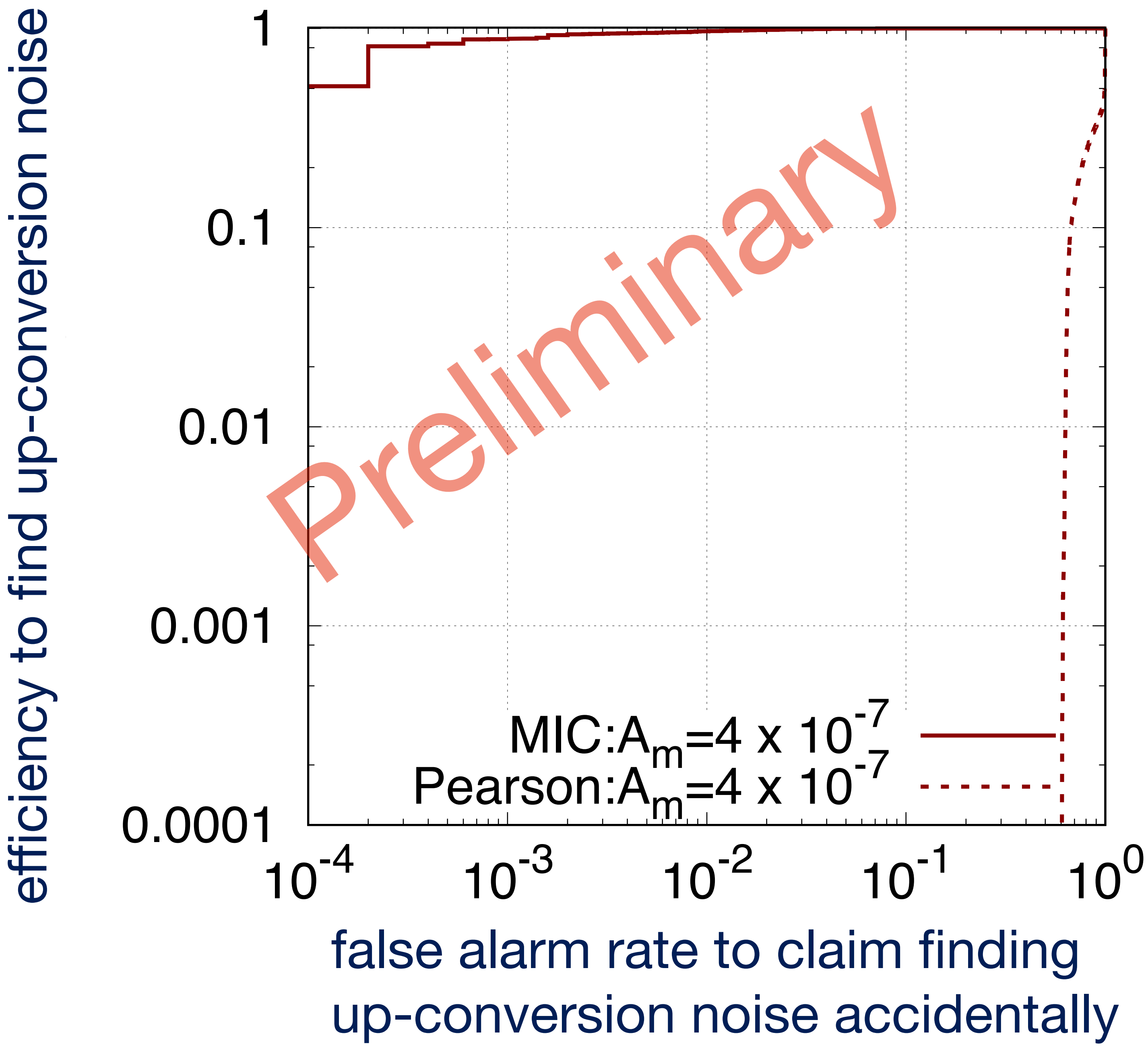
Evaluated performance - ROC curve

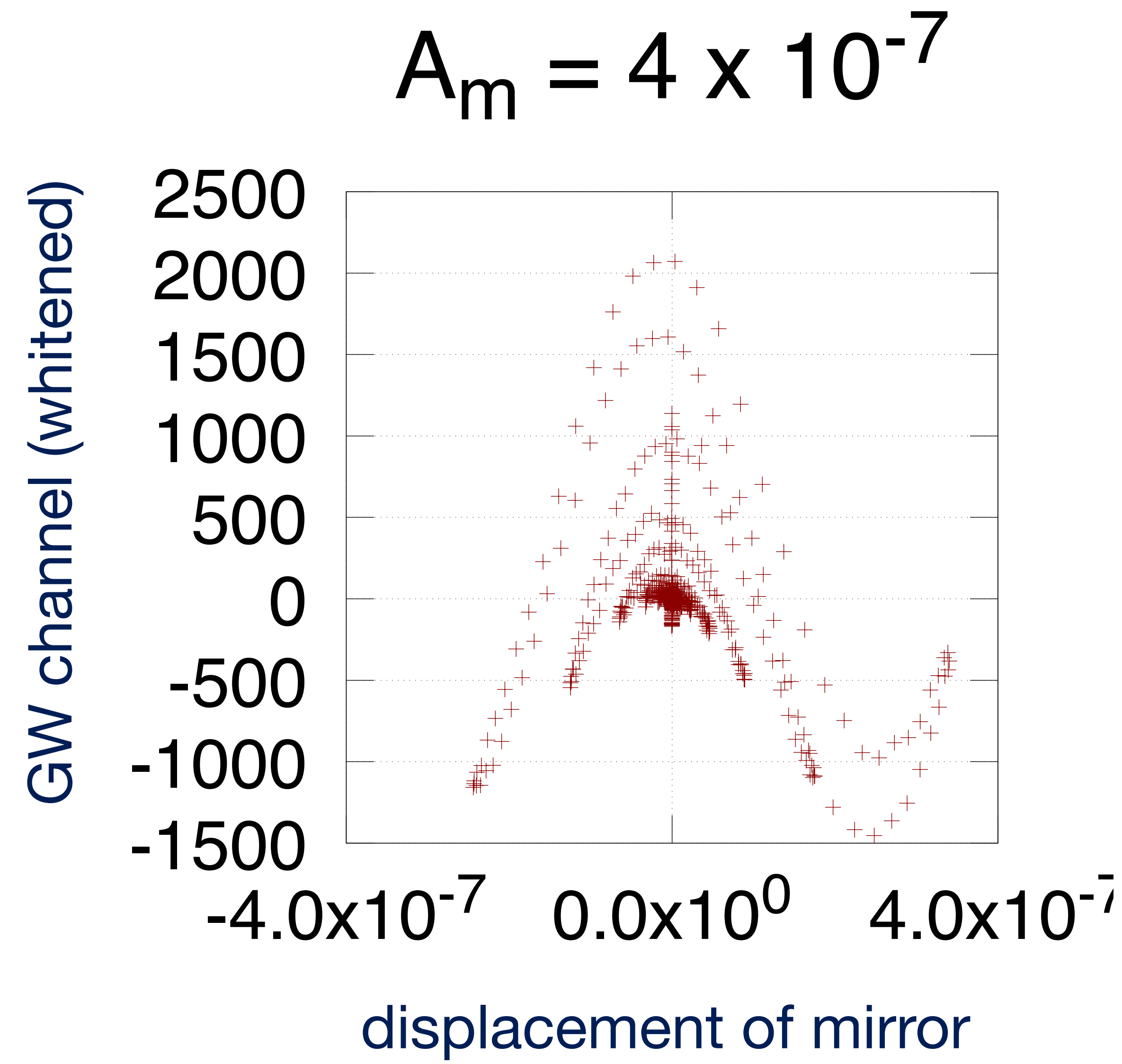




In the case of **Am = 2x10−8**, ROC curves show that Pearson and MIC have better performance, because scatter plot is linearly distributed.

Evaluated performance - ROC curve





In the case of **Am = 4x10−7**, ROC curves show that MIC detect non-linear correlation; Pearson can not detect the correlation.

Summary

o In the past operations of GW detectors, the correlated noise between multiple channels participated in preventing the achievement of the design sensitivity. The unknown noise still remains.

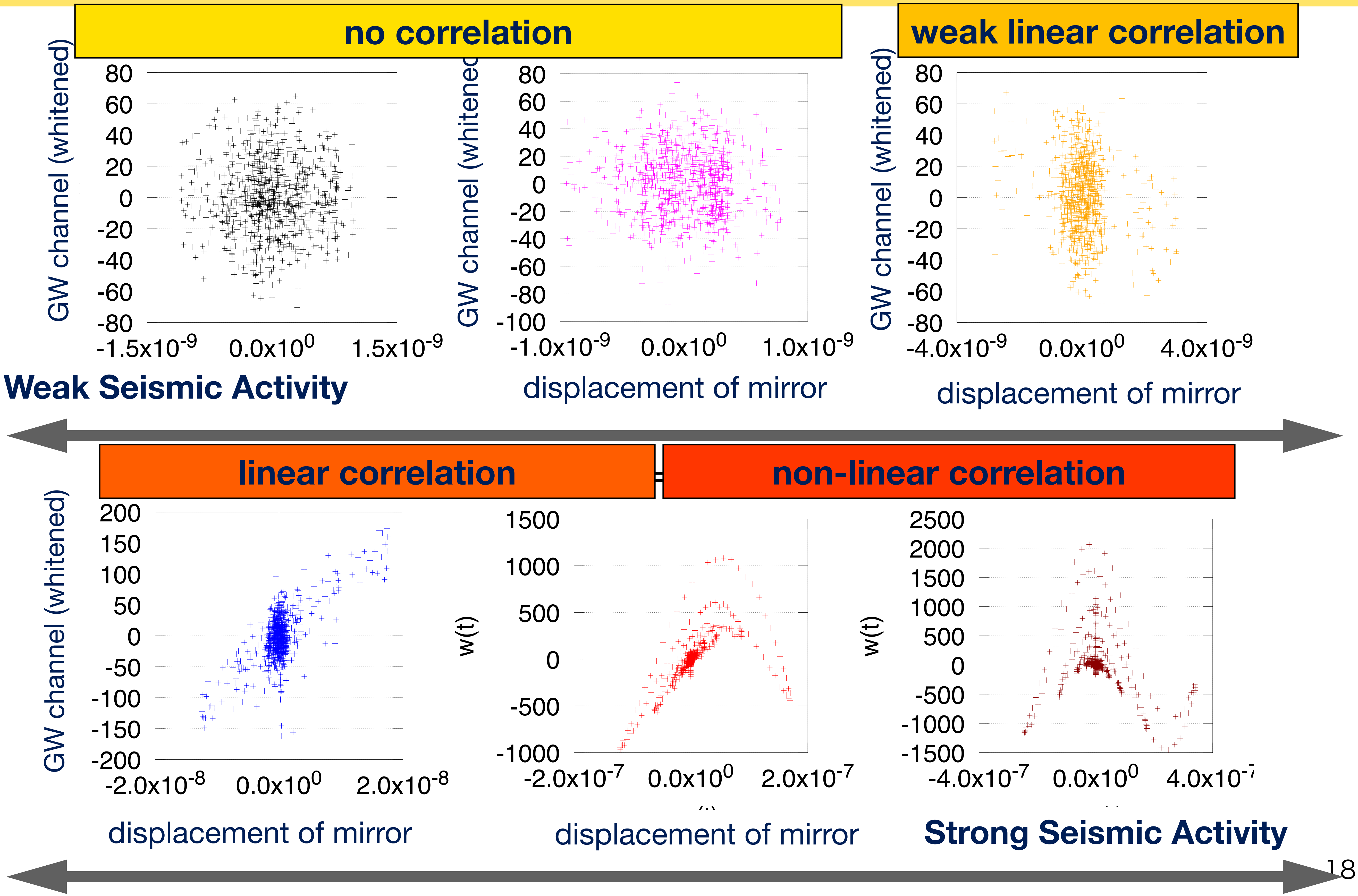
o We propose the used of the maximal information coefficient(MIC) to find non-linear correlation as well as linear correlation.

o The calculated ROC curves show that MIC can find the non-linearly correlated noise while Pearson can not detect.

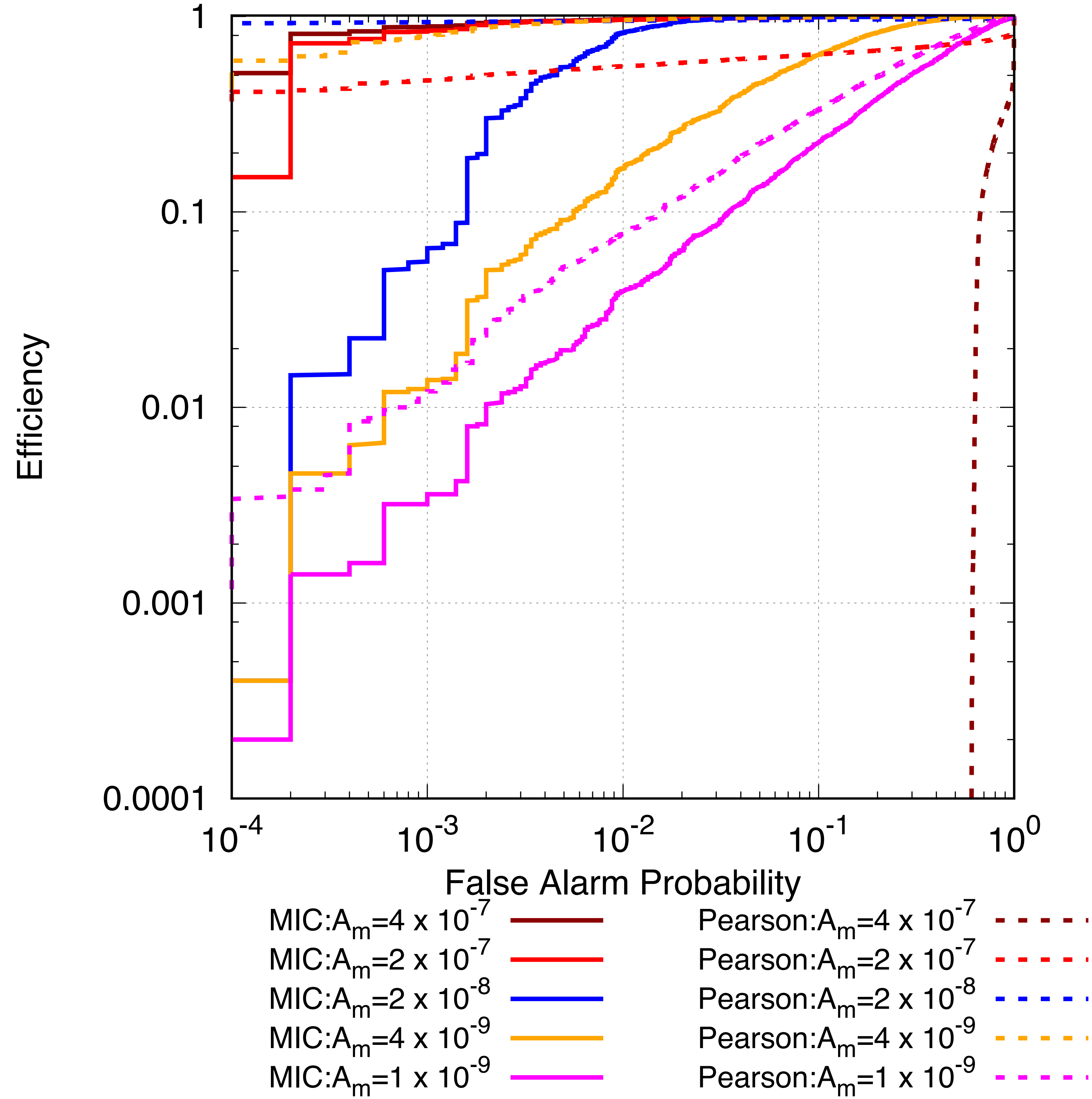
o In the commissioning phase of GW detectors,
the MIC will be useful to reveal the unknown noise, especially non-linearly correlated noise. As a result, we can

- improve the sensitivities of the detectors and
- identify false trigger event generated by GW search pipeline
  => the contribution to increase GW detection efficiency

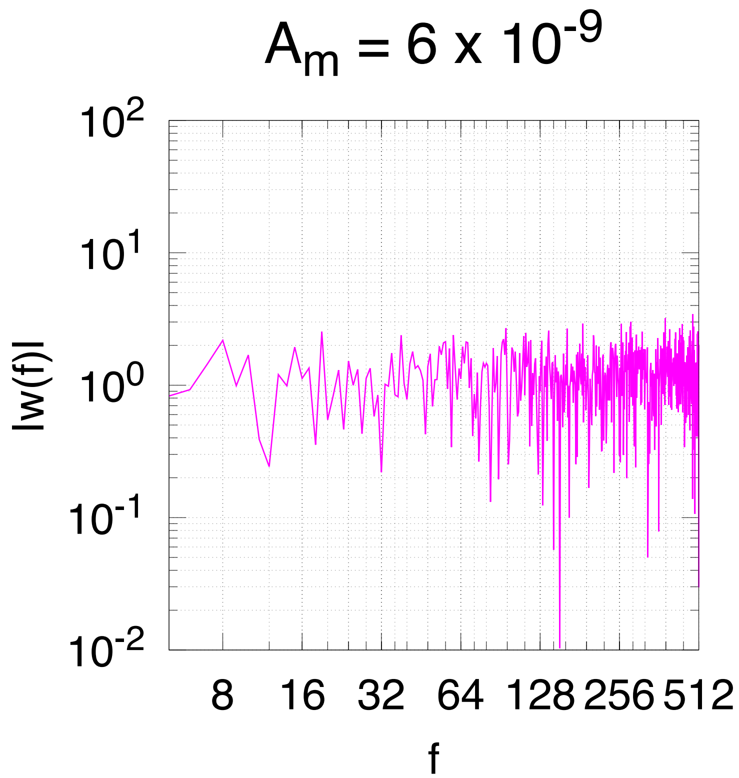
Categorization of relationship “by eye”

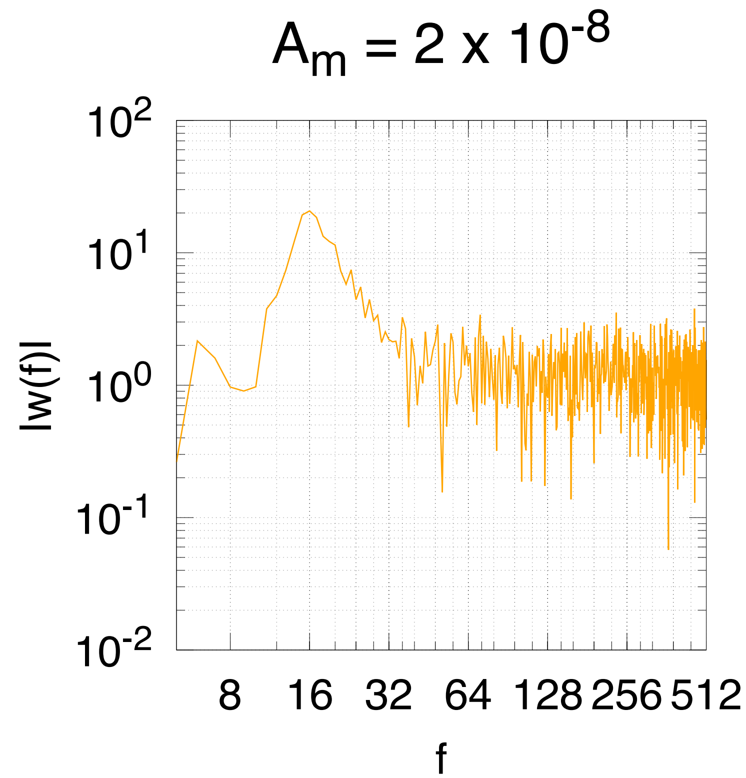


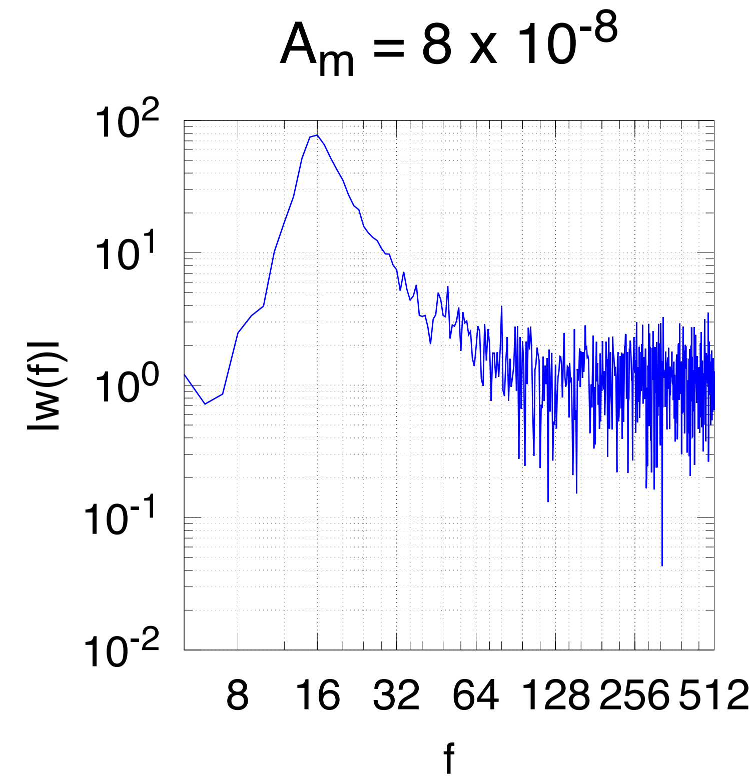
all ROC curves

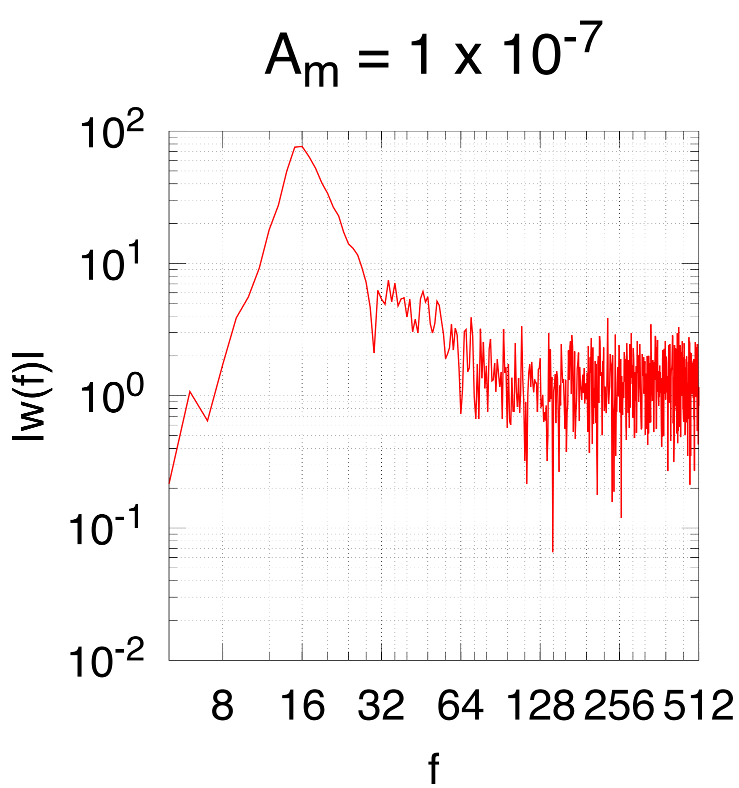


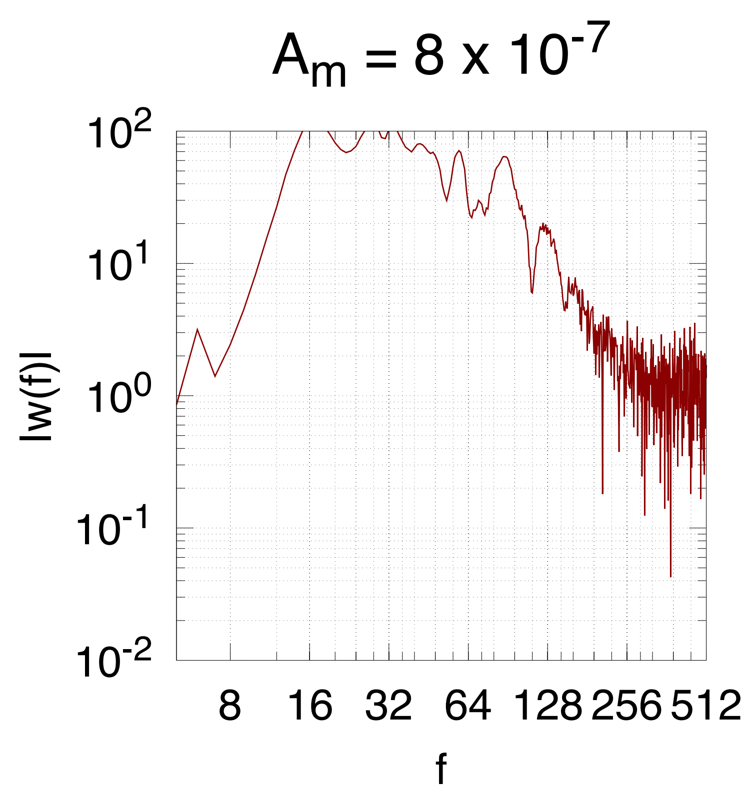
Spectrum of the simulated noise (whitened)





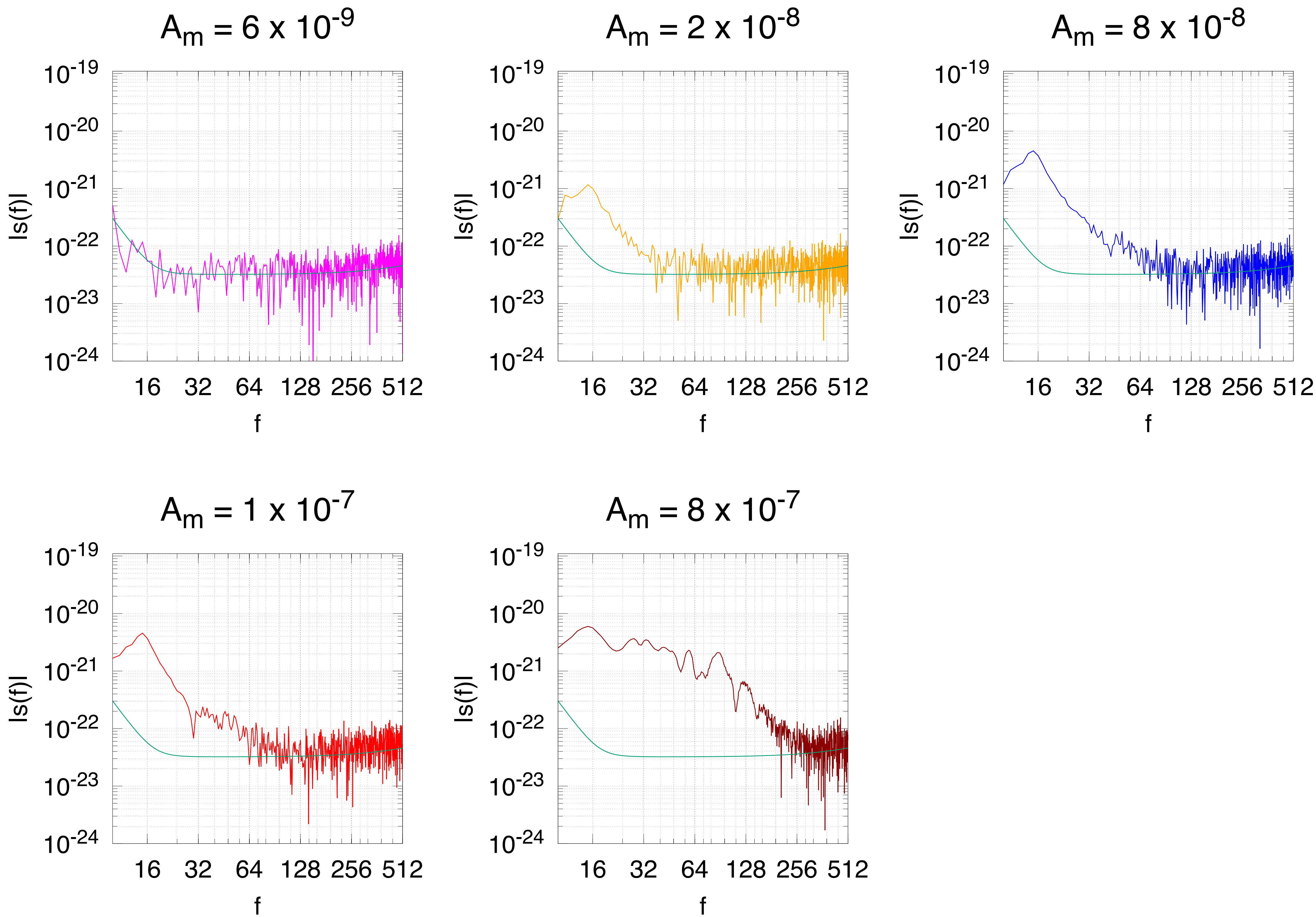




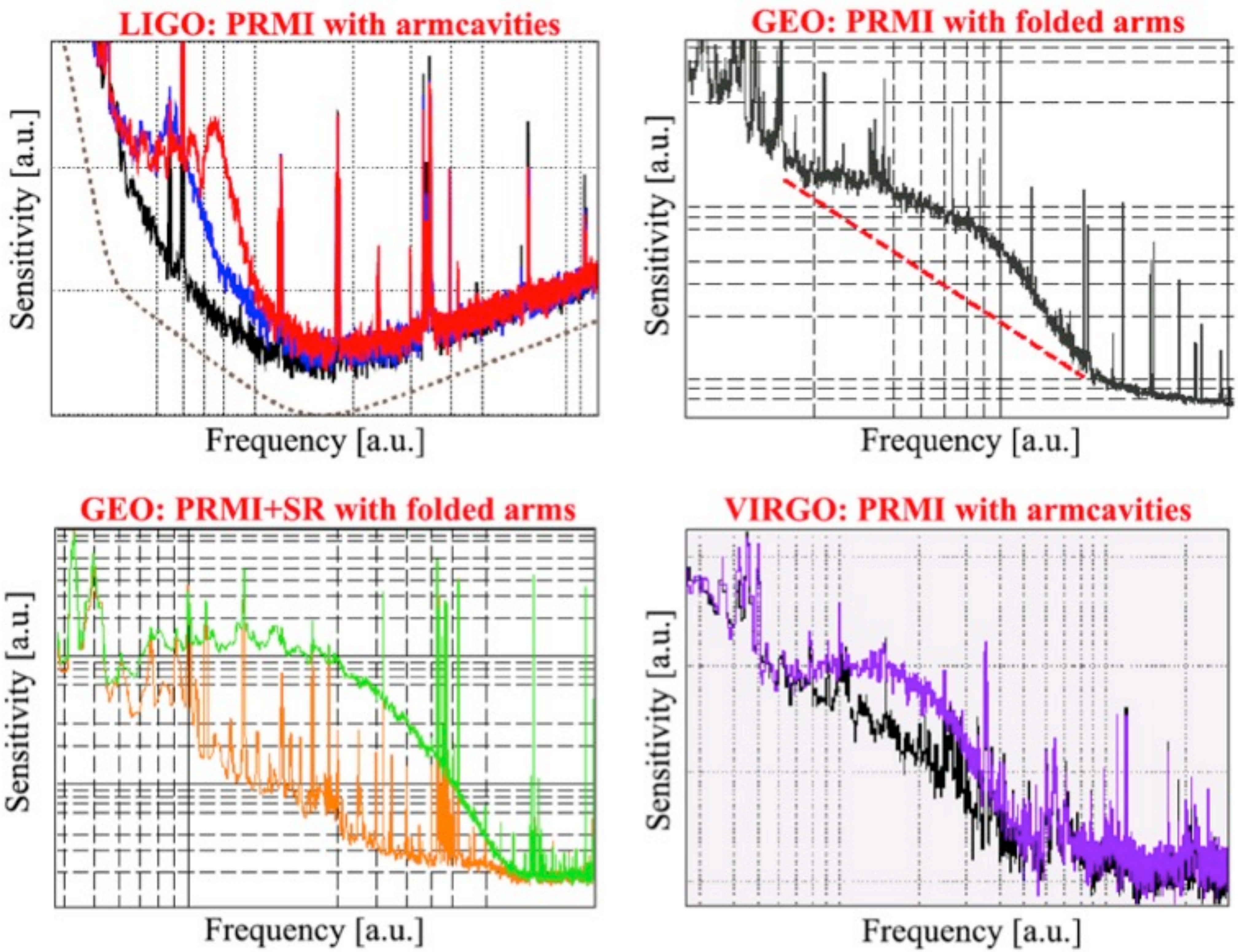


$$w(t) = \int \frac{\tilde{s}(f)}{\sqrt{S_n(f)}} e^{2\pi f t} df,$$

Spectrum of the simulated noise



example of scattered light noise



[Stefan Hild Ph.D thesis (2007)]

Strategy to apply MIC

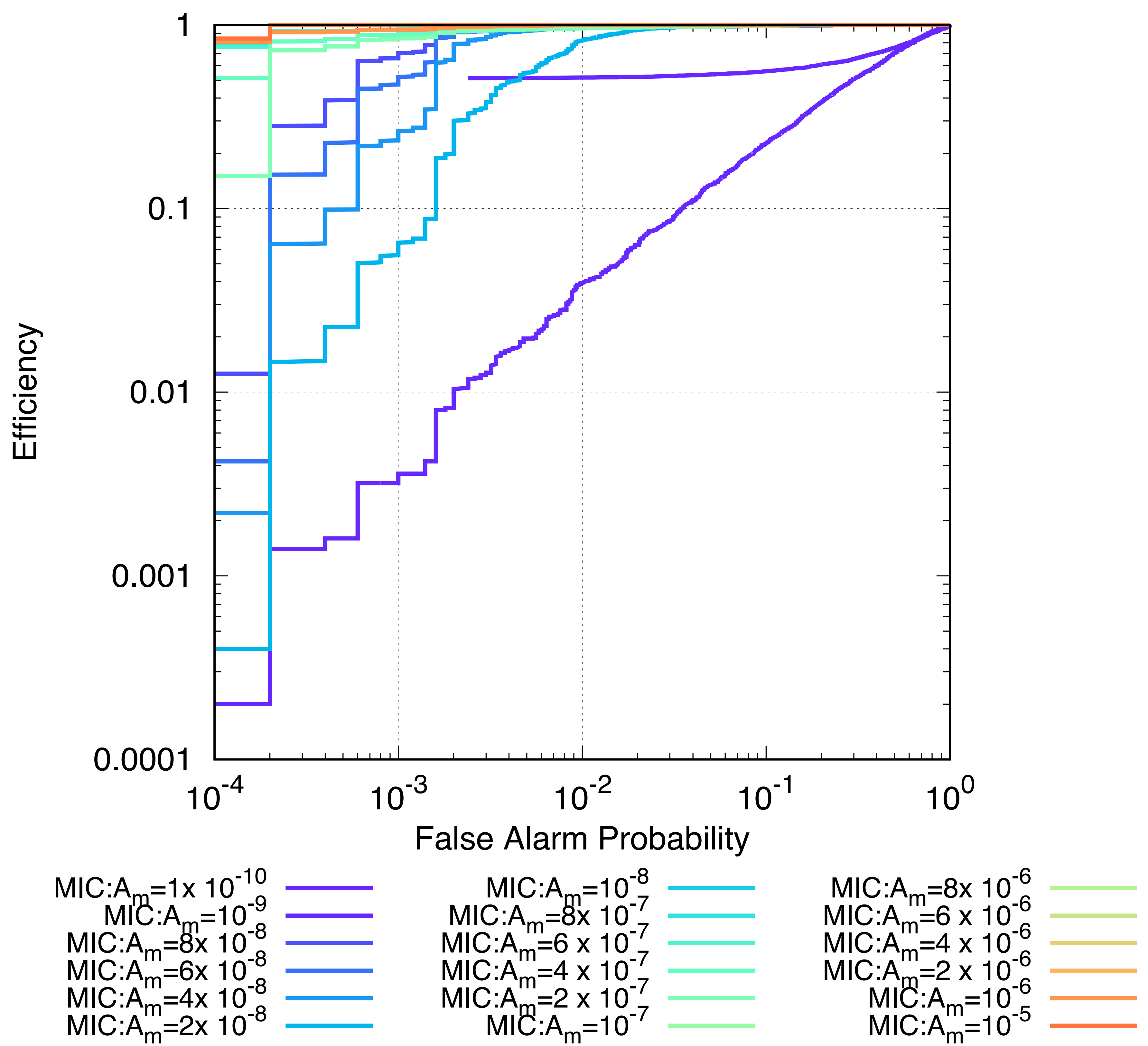
The computational cost of MIC is so heavy that we cannot apply the MIC to all channels because the many partition set are calculated.
In our proposal, the glitch search is useful to reduce the computational cost of MIC.

1. perform glitch search such as excess power and identify the transient signals

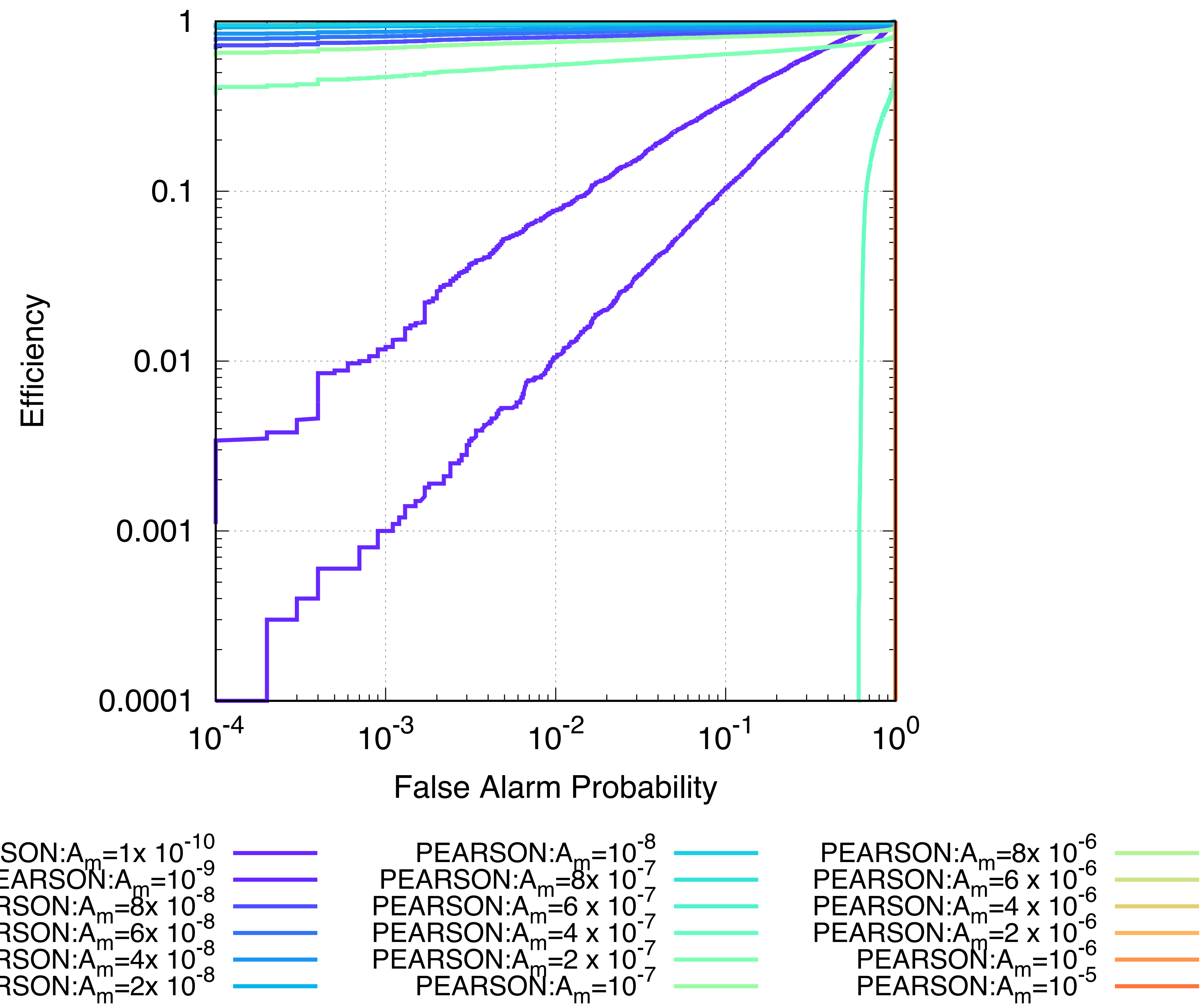
2. if the two channels are coincident, apply MIC to the two channels

3.

ROC curve(MIC)

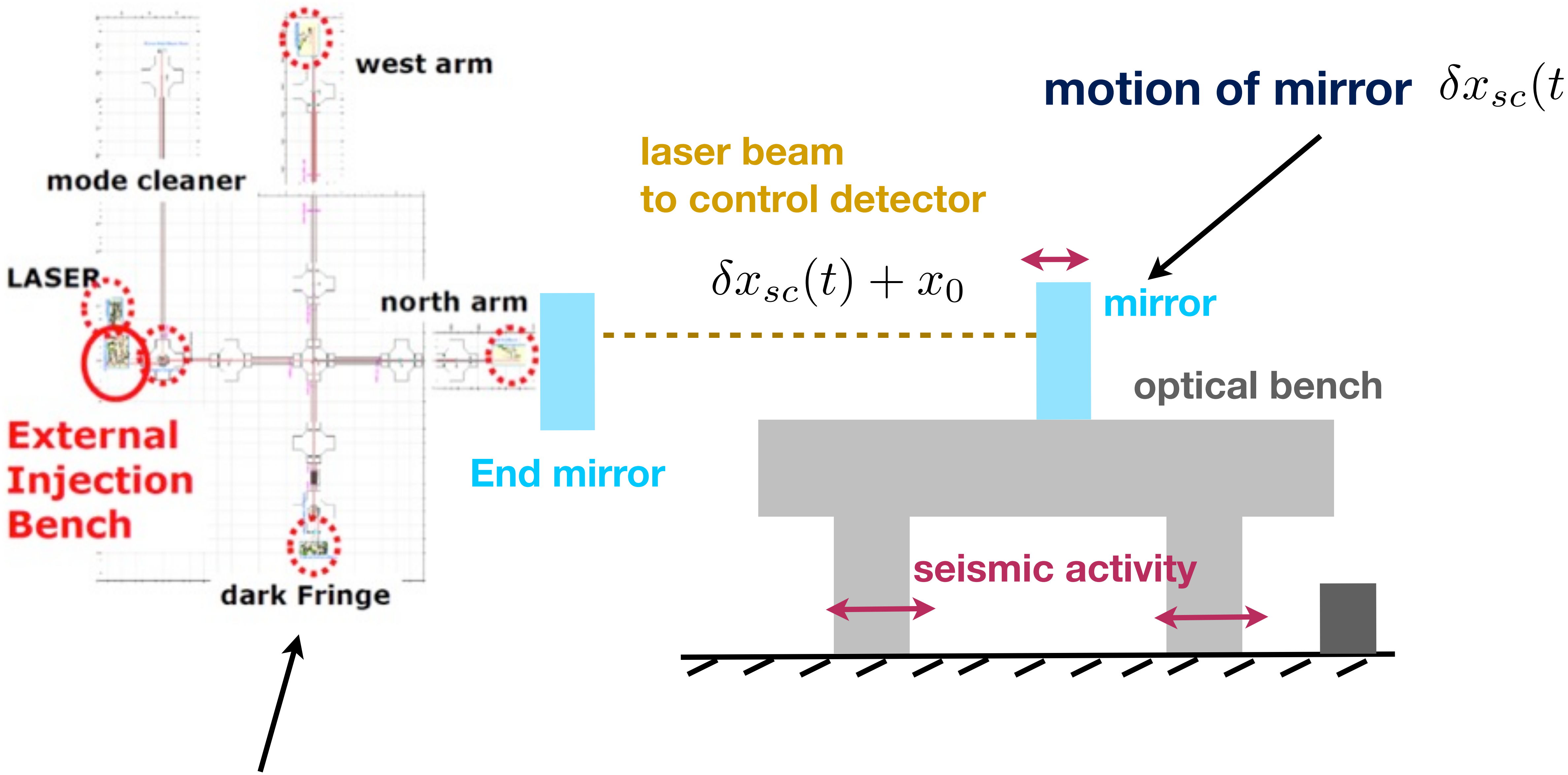


ROC curve(Pearson)



Mechanism of the up-conversion

[Classical and Quantum Gravity 27, 19 (2010) 194011]

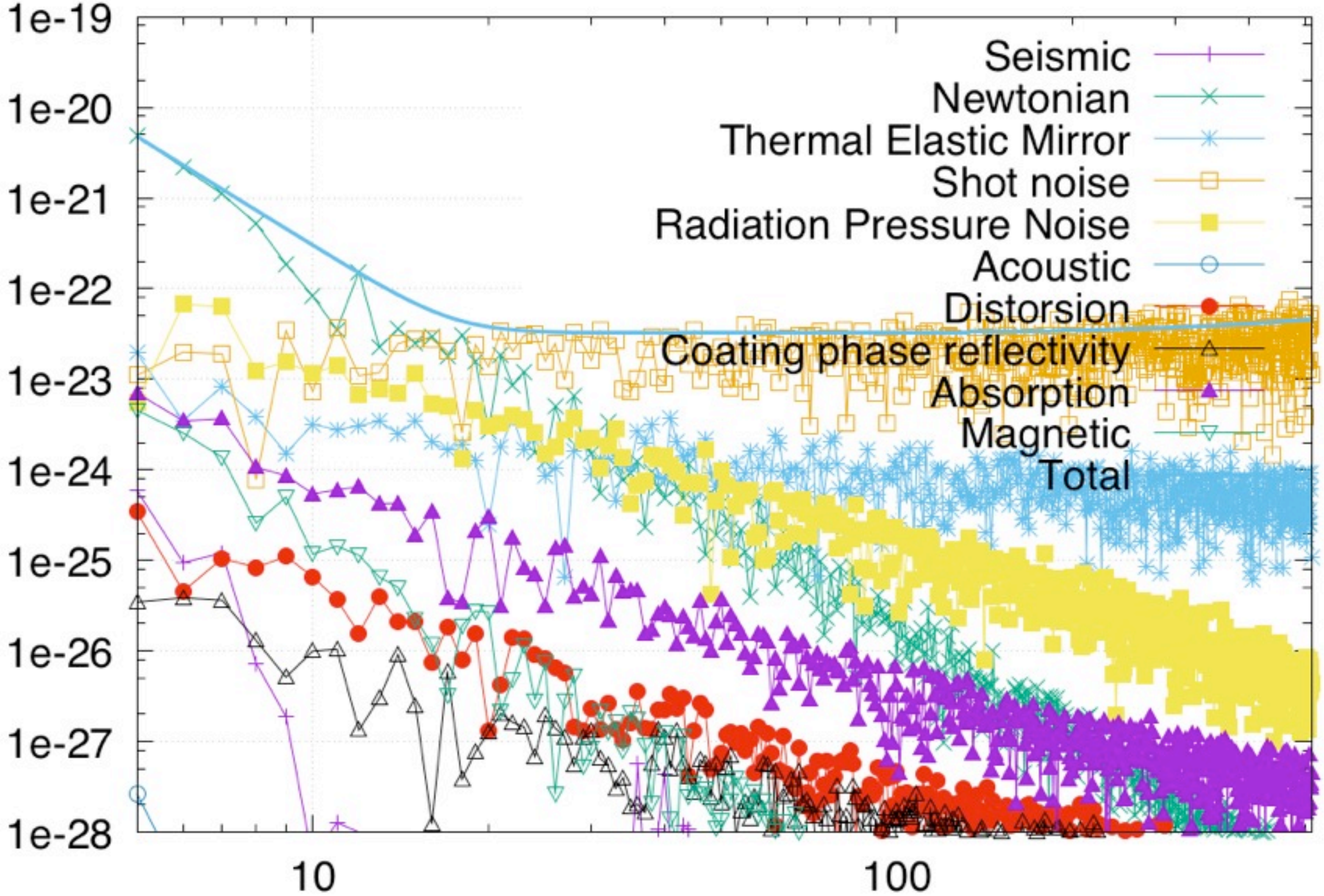


GW channel

$$s(t) = n(t) + n_{sc}(t)$$

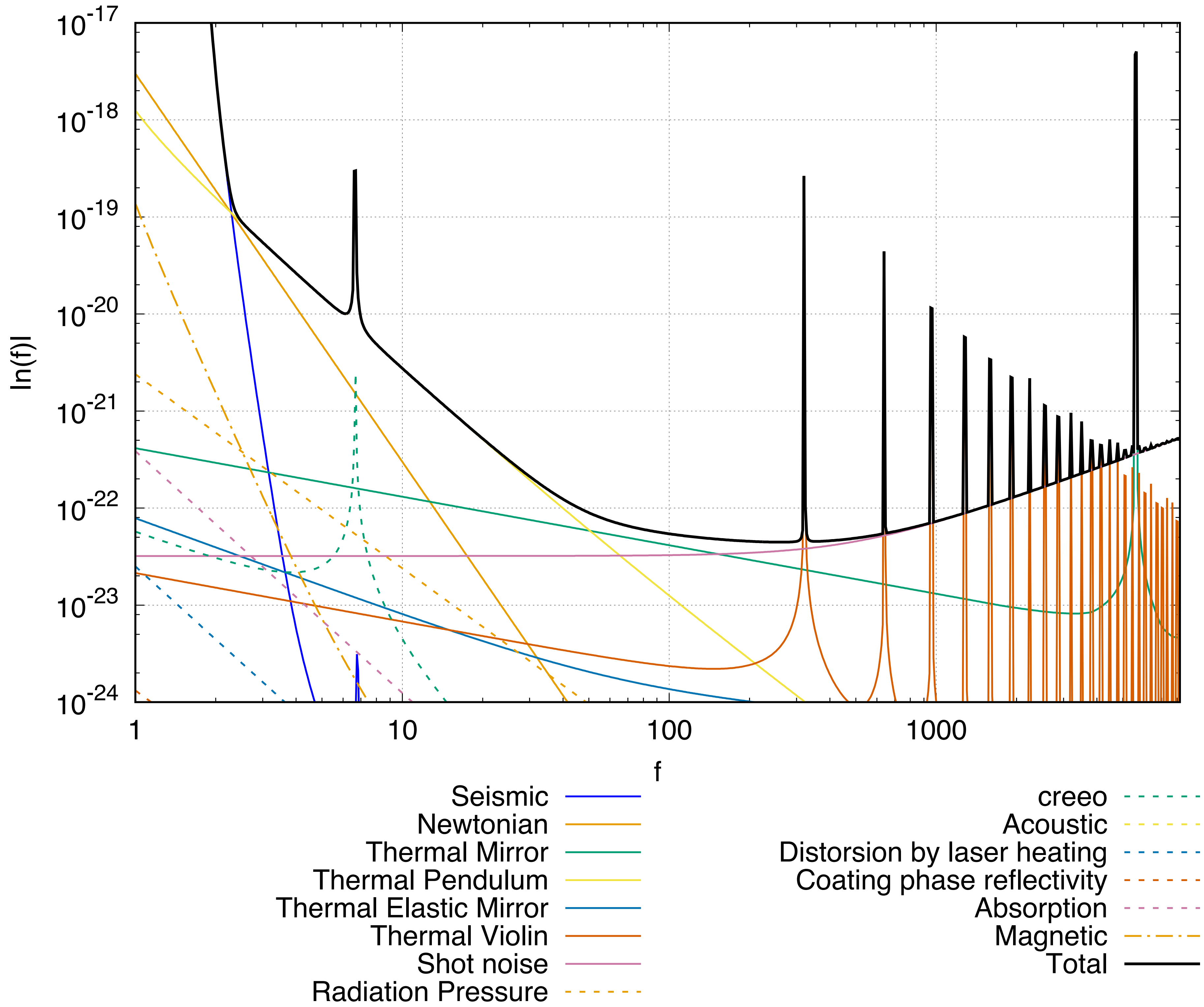
$$= n(t) + G \sin\left[\frac{4\pi}{\lambda}(x_0 + \delta x_{sc}(t))\right]$$

Virgo sensitivity curve used in this study

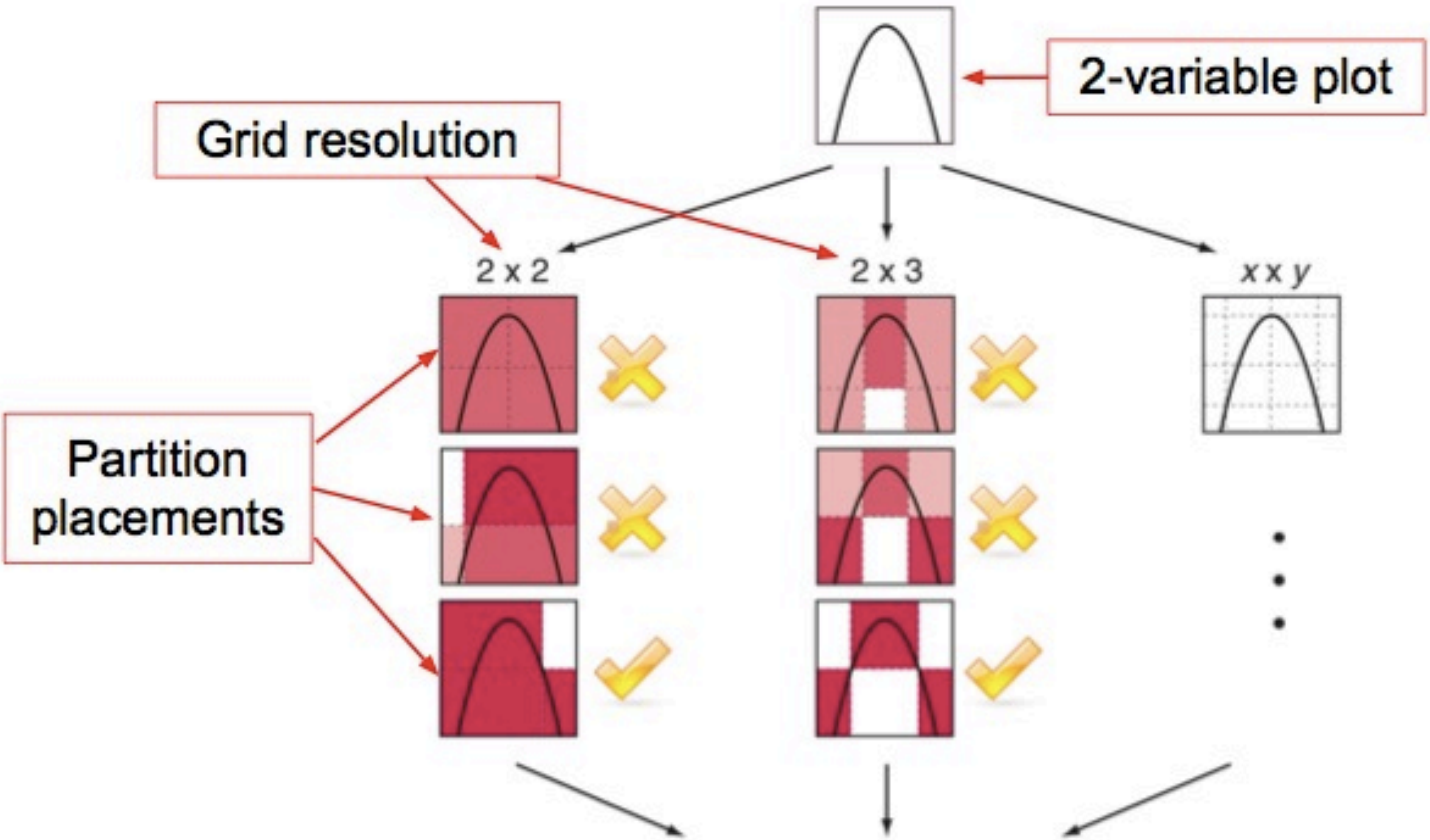


Virgo sensitivity curve

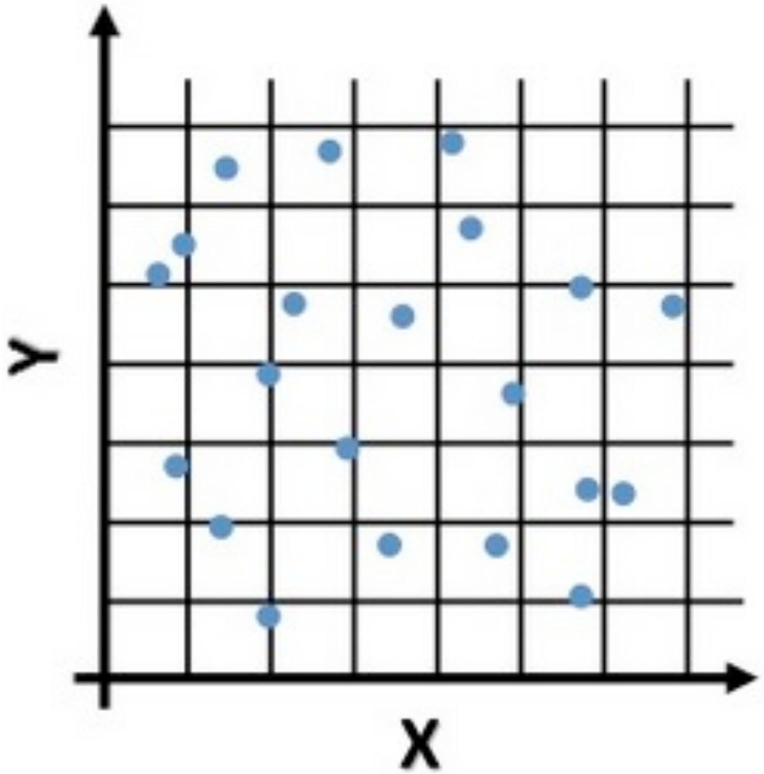
http://w3-1.virgo-gw.eu/senscurve/



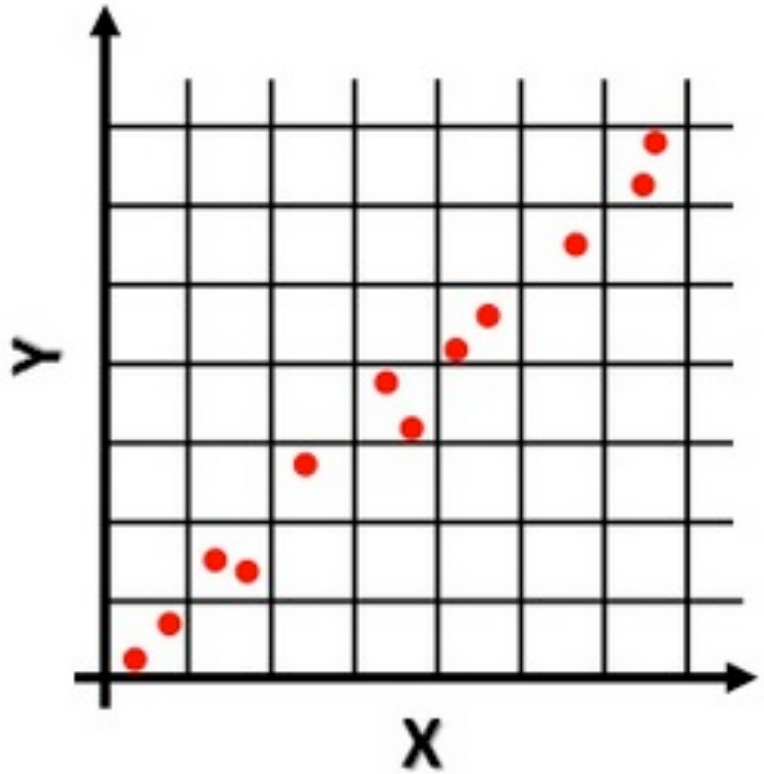
grid partition



Maximum Information Coefficient (MIC)



No correlation



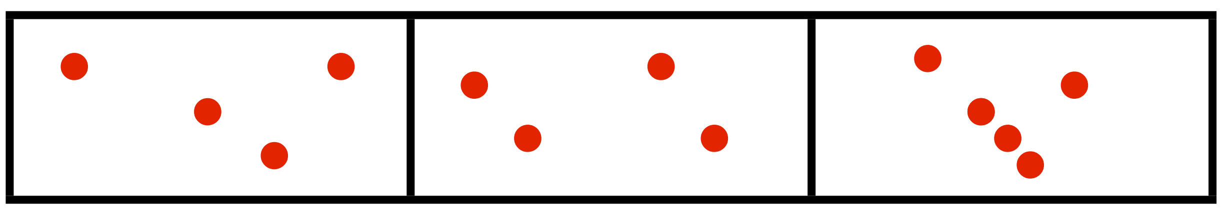
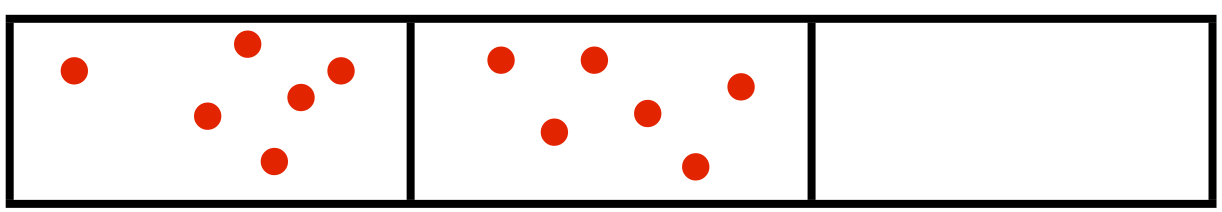
Linear correlation

The concept of MIC is that
if a relationship exists between two channels, grids can be drawn on the scatter plot of two data that partitions the data to catch that relationship.

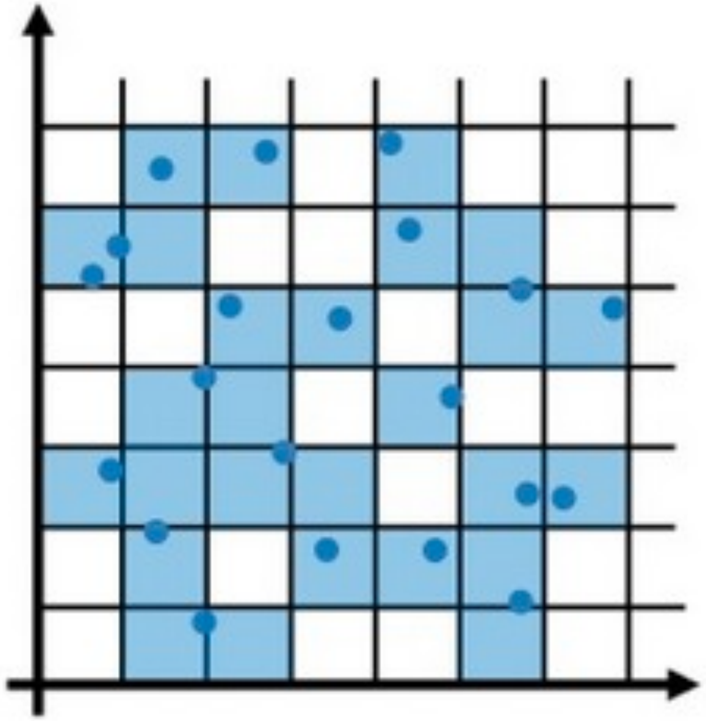
http://www.slideshare.net/logics-of-blue/mic-31810194

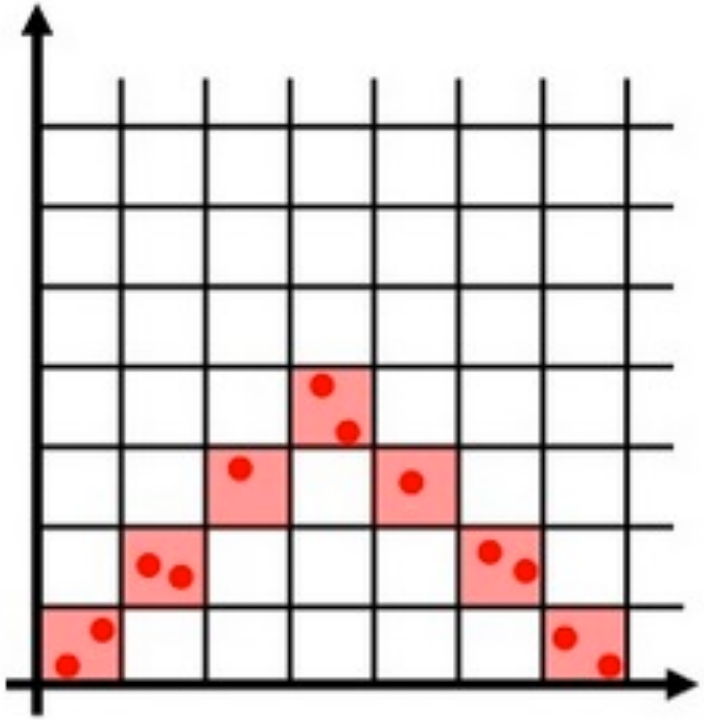
Maximum Information Coefficient (MIC)

Mutual information quantifies how much low the uncertainty of the data is.

example	uncertainty	mutual information
① 	high	low
② 	middle	middle
③ 	low	high

Maximum Information Coefficient (MIC)



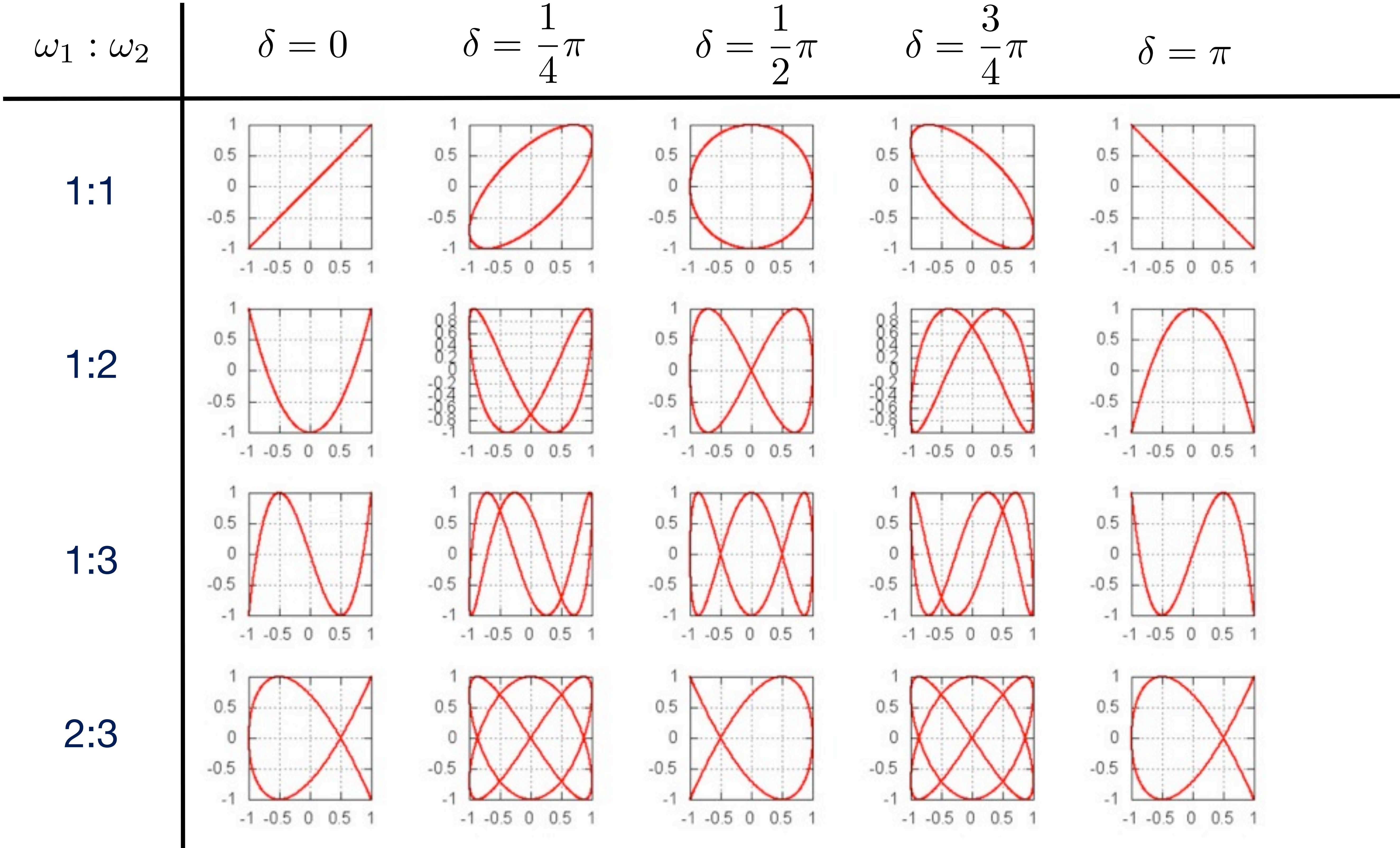


looks like no correlation
=> low mutual information
=> we conclude no correlation

data concentrate in seven cells
=> high mutual information
=> we conclude there is correlation

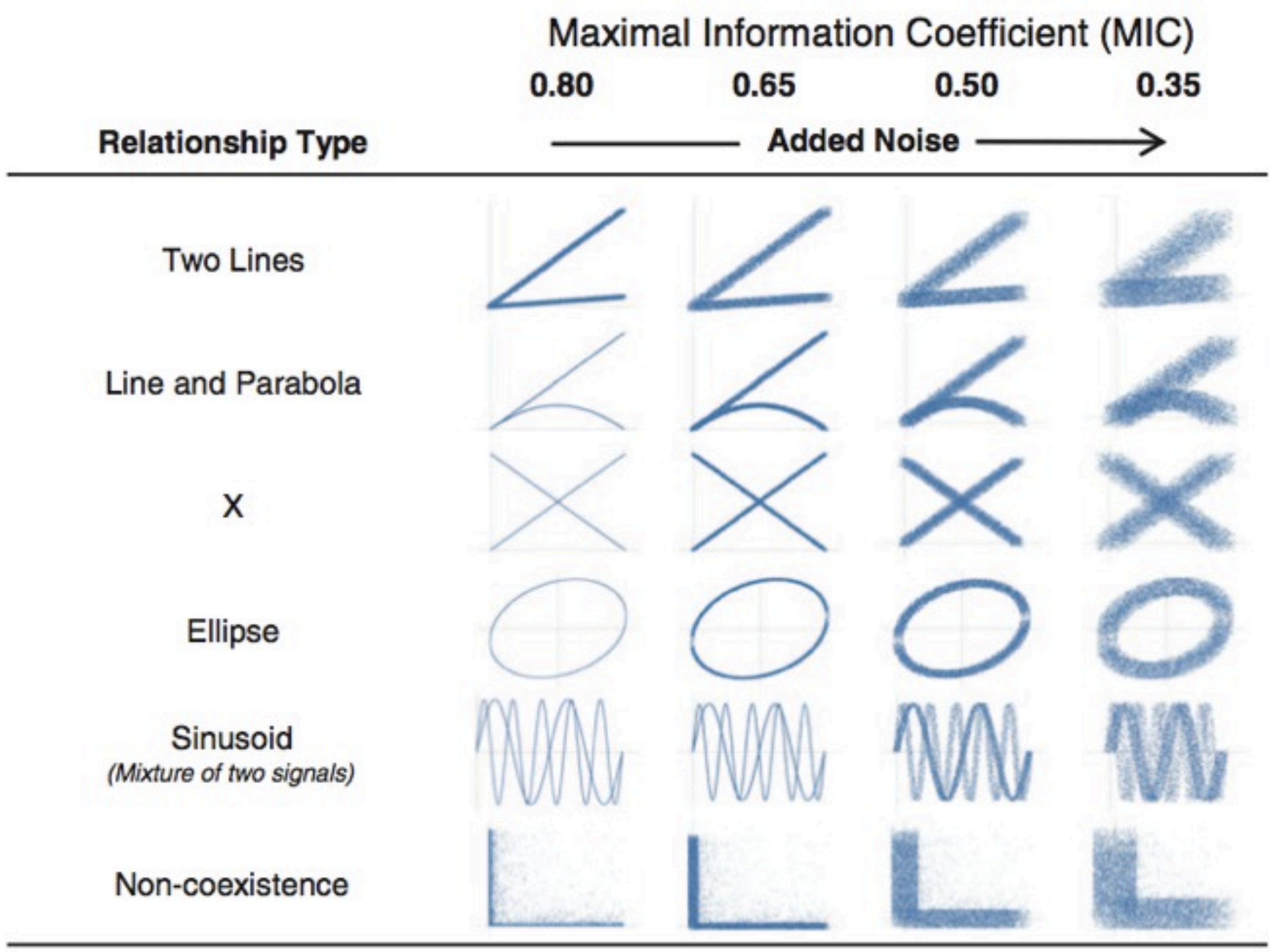
Lissajous figure

$$x = \sin(\omega_1 t)$$
$$y = \sin(\omega_2 t + \delta)$$



Which correlation MIC can find?

o MIC can find a non-linear correlation as well as linear correlation.



[David N. Reshef, et al. Science 334, 1518 (2011)]

optical bench motion caused by seismic noise of Virgo

Performance

The transfer function of our seismic attenuation system, in a controlled state, has been measured. Using the typical ground motion spectrum at Virgo, the bench motion, as it would be at Virgo, can be calculated. It is shown in Fig. 7 (red curve), together with the requirements for Advanced Virgo (black dotted curve) and the motion of the bench used in Virgo+ (blue curve). Clearly, our system meets the requirements for all frequencies we can measure.

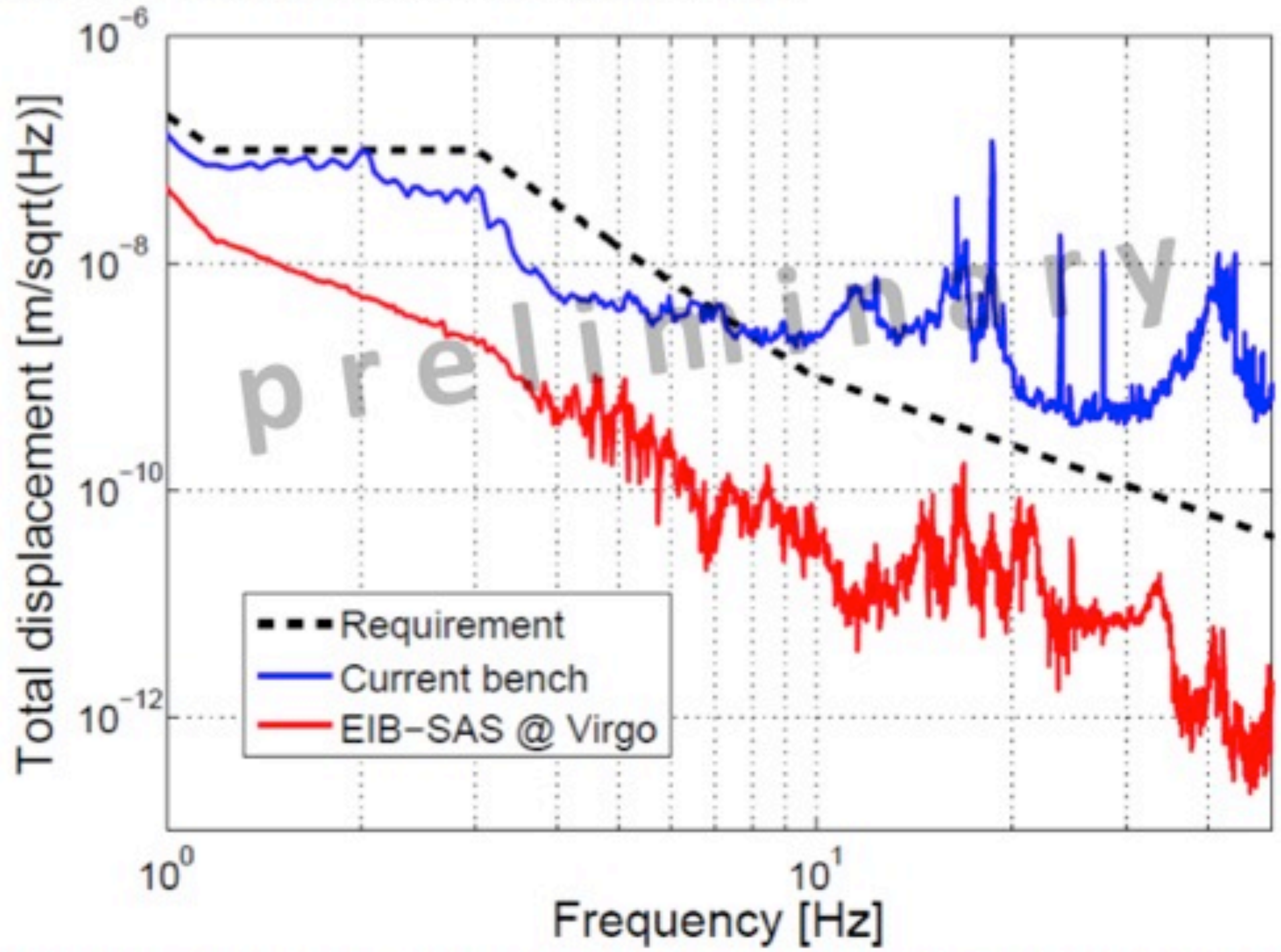


Figure 7: Displacement spectrum of the bench (red curve), with the displacement spectrum of the bench used for Virgo+ (blue curve) and the requirement for Advanced Virgo (black, dotted line.)

http://www.nikhef.nl/pub/departments/mt/projects/virgo/general/posters/Nikhef_Poster_Virgo_EIB_SAS.pdf