

Local Hurst exponent computation of data from triaxial seismometers monitoring Kagra

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Outline

- Summary of Data Analysis and Fathon Algorithm

Methodology

- Persistency of a time series and the Hurst exponent H
- Detrended Fluctuation Analysis (DFA)
- Local Hurst Exponent $H(t)$

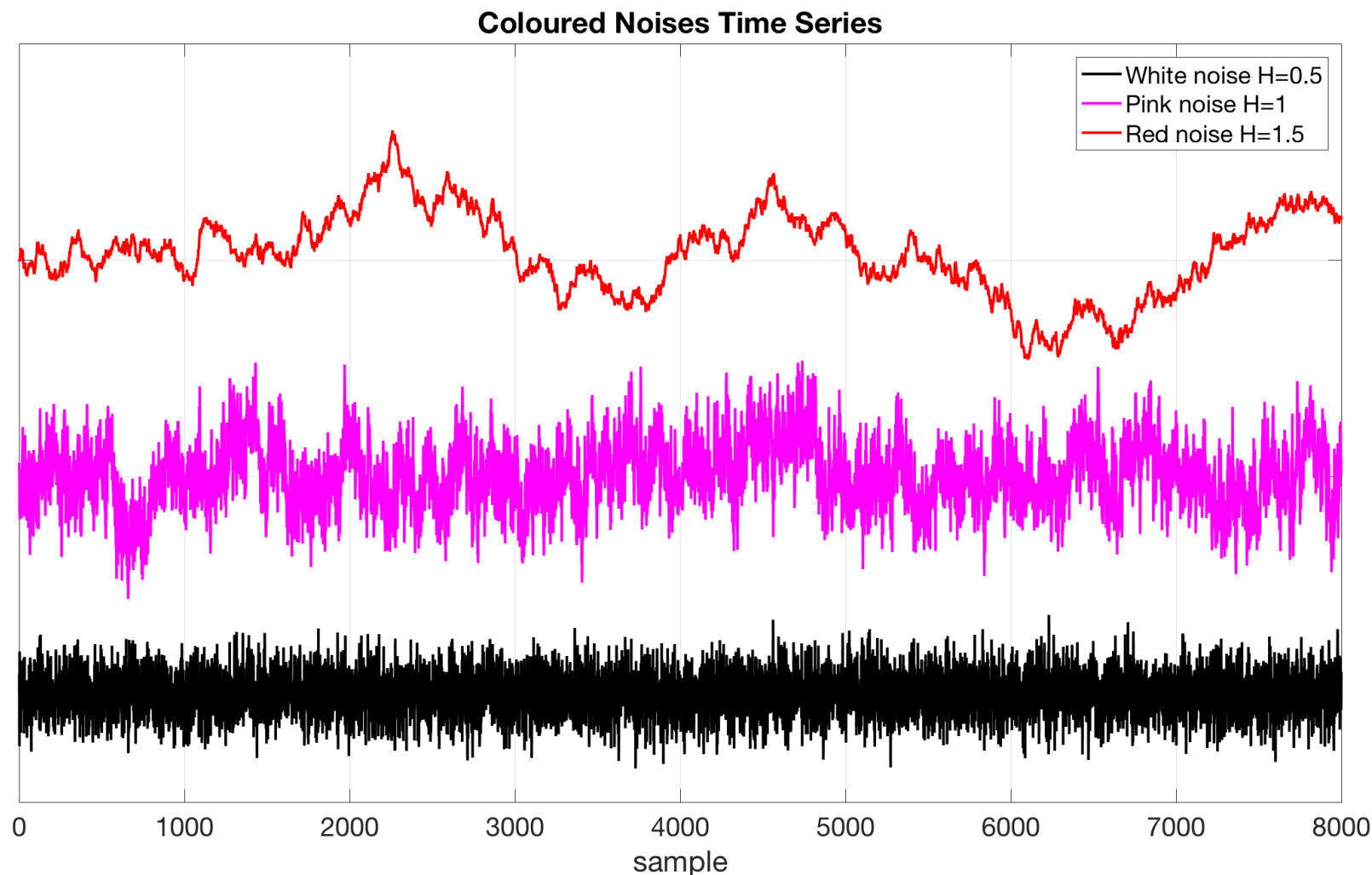
Results

- KAGRA Triaxial Seismometers Data Analysis
- EXTRA SLIDES: Applications to Virgo Data

Summary of Data Analysis and Fathon Algorithm

- **Data analysis:** The local Hurst exponent $H(t)$ was computed for three triaxial seismometers monitoring the KAGRA central room and the two end buildings of the interferometer, to quantify data persistency.
- **Algorithm:** The analysis was carried out using **fathon**, an open source software for fast fractal analysis in Python (Stefano Bianchi, PhD)
- **Reference:** S. Bianchi, “fathon: A Python package for a fast computation of detrended fluctuation analysis and related algorithms,” Journal of Open Source Software, vol. 5, no. 45, p. 1828, 2020.
- <https://pypi.org/project/fathon/>

Hurst Exponent to Quantify Time Series Persistency

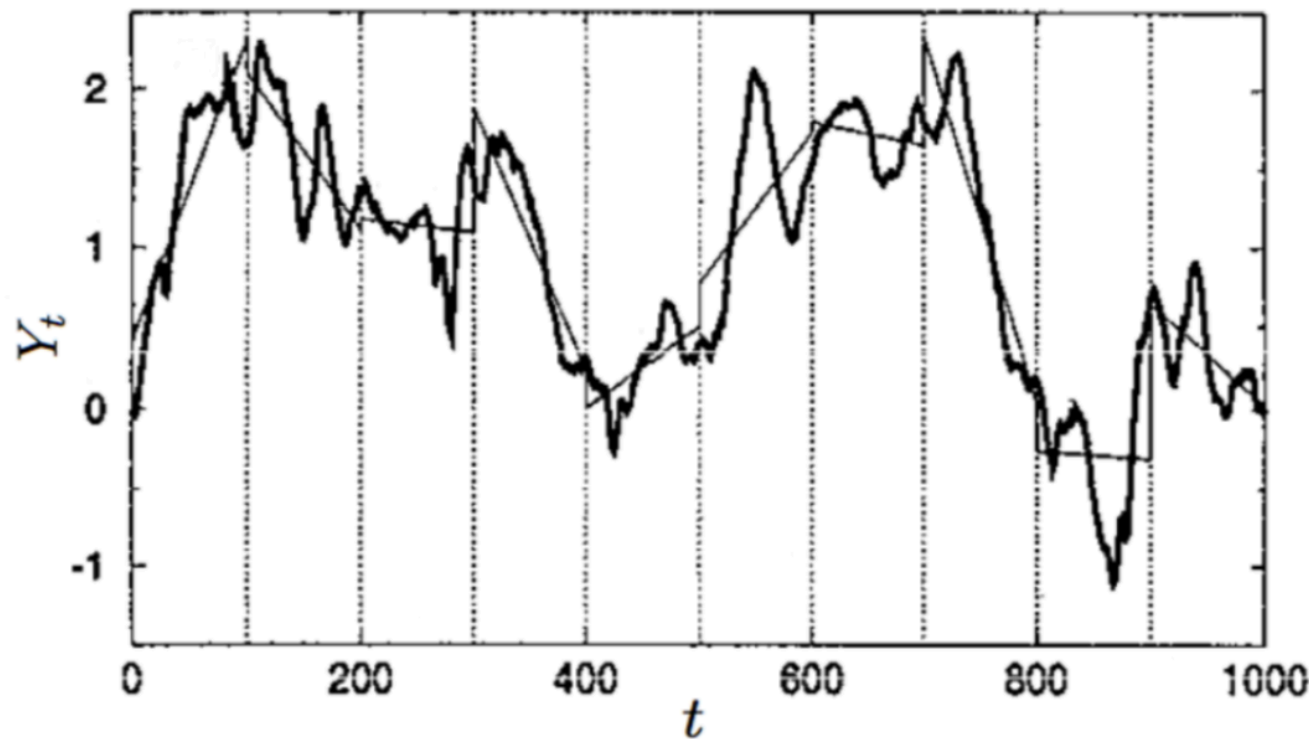


- **Red noise: $H=1.5$, ($\beta=2$)**
- **Pink noise: $H=1$, ($\beta=1$)**
- **White noise: $H=0.5$, ($\beta=0$)**

- H : quantify time series persistency (speed of oscillations around the mean)
- Spectra of power law process $P(f) \sim f^{-\beta}$
- β “spectral index” ($\beta = 0 \rightarrow$ flat spectra \rightarrow white noise)
- Relation with spectral index

$$H = \frac{\beta + 1}{2}$$

1. Detrended Fluctuation Analysis (DFA)



- Figure adapted from: "Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series," Chaos: An Interdisciplinary Journal of Nonlinear Science, vol. 5, no. 1, pp. 82–87, 1995

In this Figure

- $n=100$ (window length), $N=1000$ (data length)
- $N_{tot}=10$ (number of windows)
- $s=1 \dots 10$ (window index)
- A linear fit Y_s^{fit} is subtracted

- **Output of the DFA algorithm is the Hurst exponent H**
- DFA: applied to random walk like time series $Y(t)$, where $t = 1 \dots N$. If data are instead noise like $x(t)$, mean subtraction and integration is carried out first

$$Y(t) = \sum_{t'=1}^t (x(t') - \bar{x})$$

- $Y(t)$ is divided into $N_{tot} = N/n$ non-overlapping windows of length n .
- Data in each window $s = 1 \dots N_{tot}$ is fitted with a least squares line Y_s^{fit}

2. Detrended Fluctuation Analysis (DFA)

- RMS(n,s) computed in each window

$$RMS(n, s) = \sqrt{\frac{1}{n} \sum_{i=1}^n [Y((s-1)n + i) - Y_s^{fit}(i)]^2}$$

for $s = 1 \dots N_{tot}$

- **Scaling function $F(n)$:** average of squared RMS obtained for the time intervals of length n

$$F(n) = \left[\frac{1}{N_{tot}} \sum_{s=1}^{N_{tot}} RMS^2(n, s) \right]^{1/2}$$

- Compute $F(n)$ for different values of n -> power law relation with window size n
-> The exponent is the **Hurst exponent**:

$$F(n) \sim n^H$$

- **H estimation:** slope of $\log(F(n))$ vs $\log(n)$

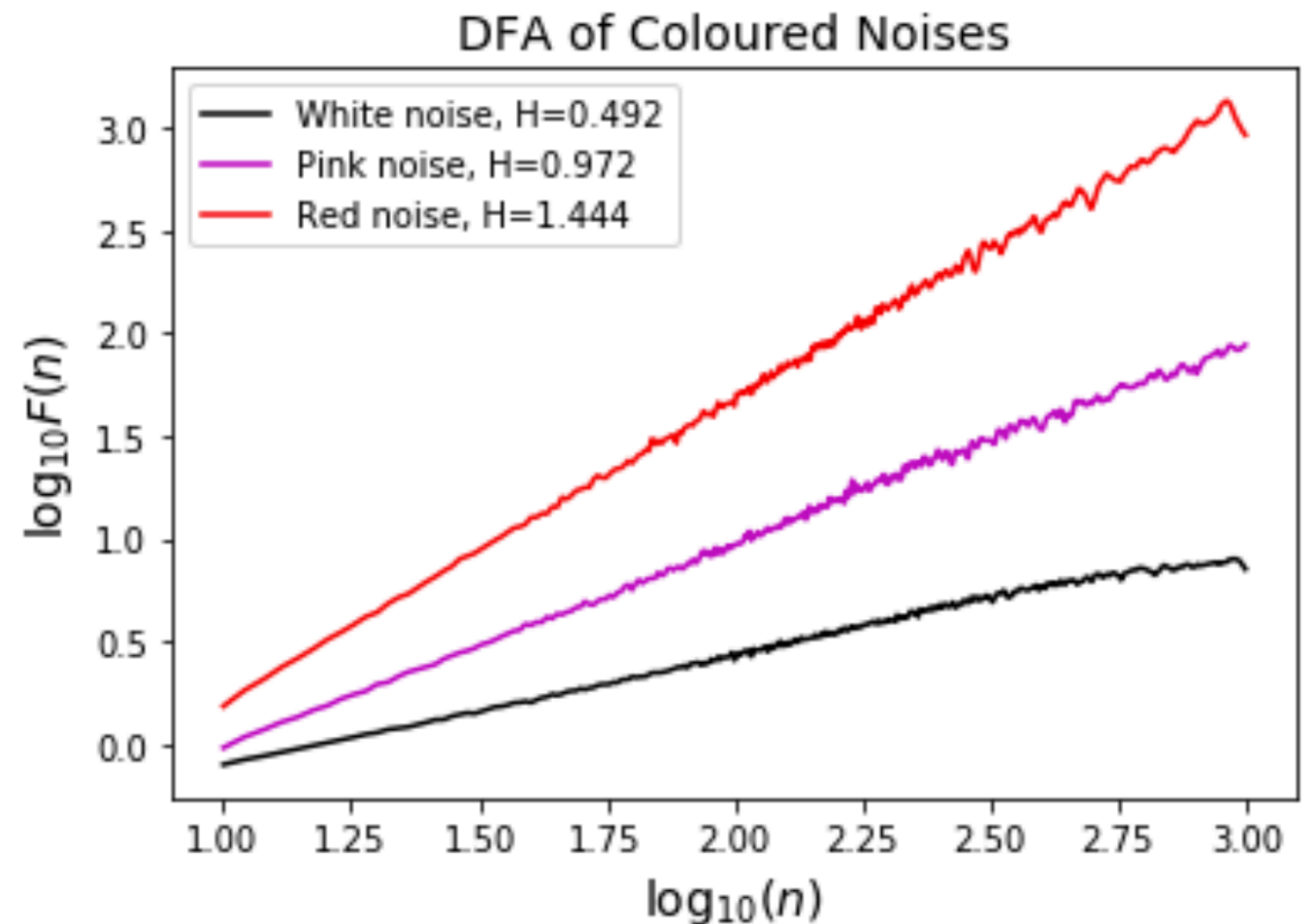
Fathon Algorithm Testing: H for Coloured Noises



- **Red noise:** $H=1.5$
- **Pink noise:** $H=1$
- **White noise:** $H=0.5$

DFA Output

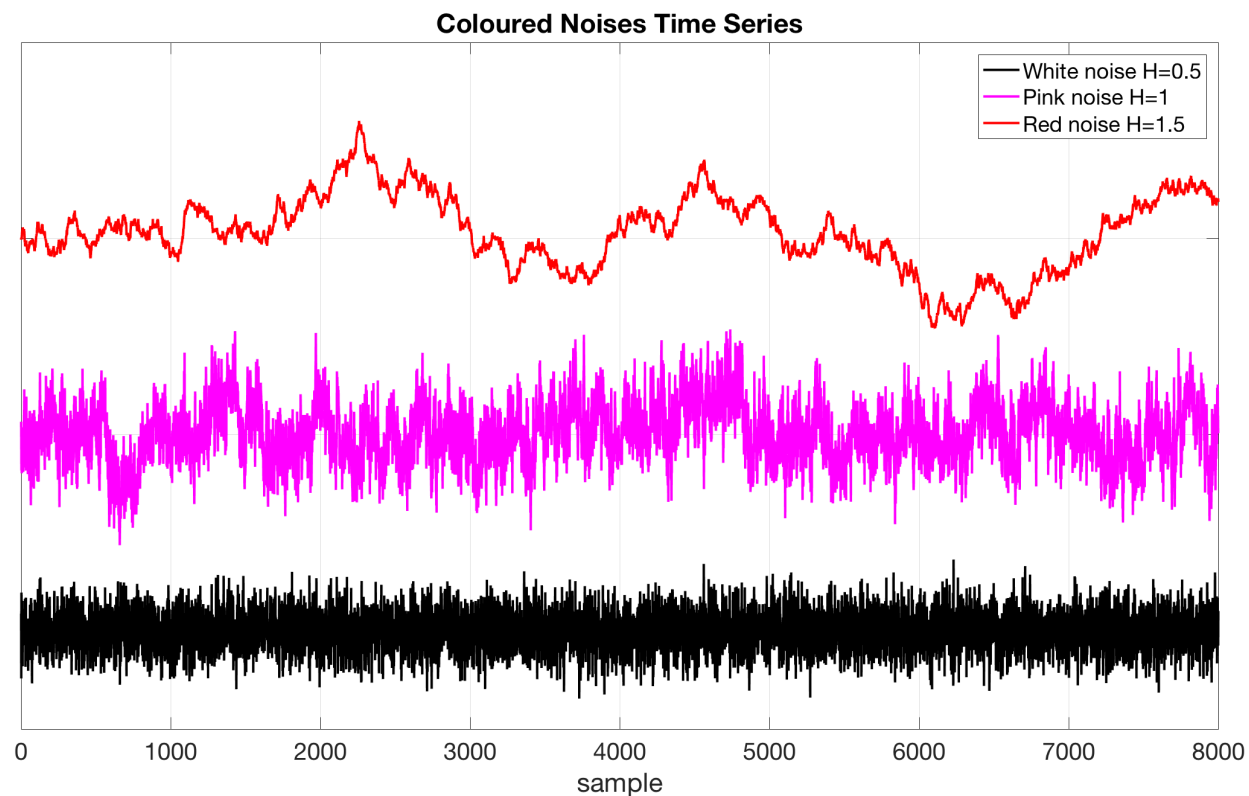
- **Red noise:** $H=1.444$
- **Pink noise:** $H=0.972$
- **White noise:** $H=0.492$
- H of coloured noises correctly estimated from slope of $\log(F(n))$ vs $\log(n)$



Local Hurst Exponent $H(t)$

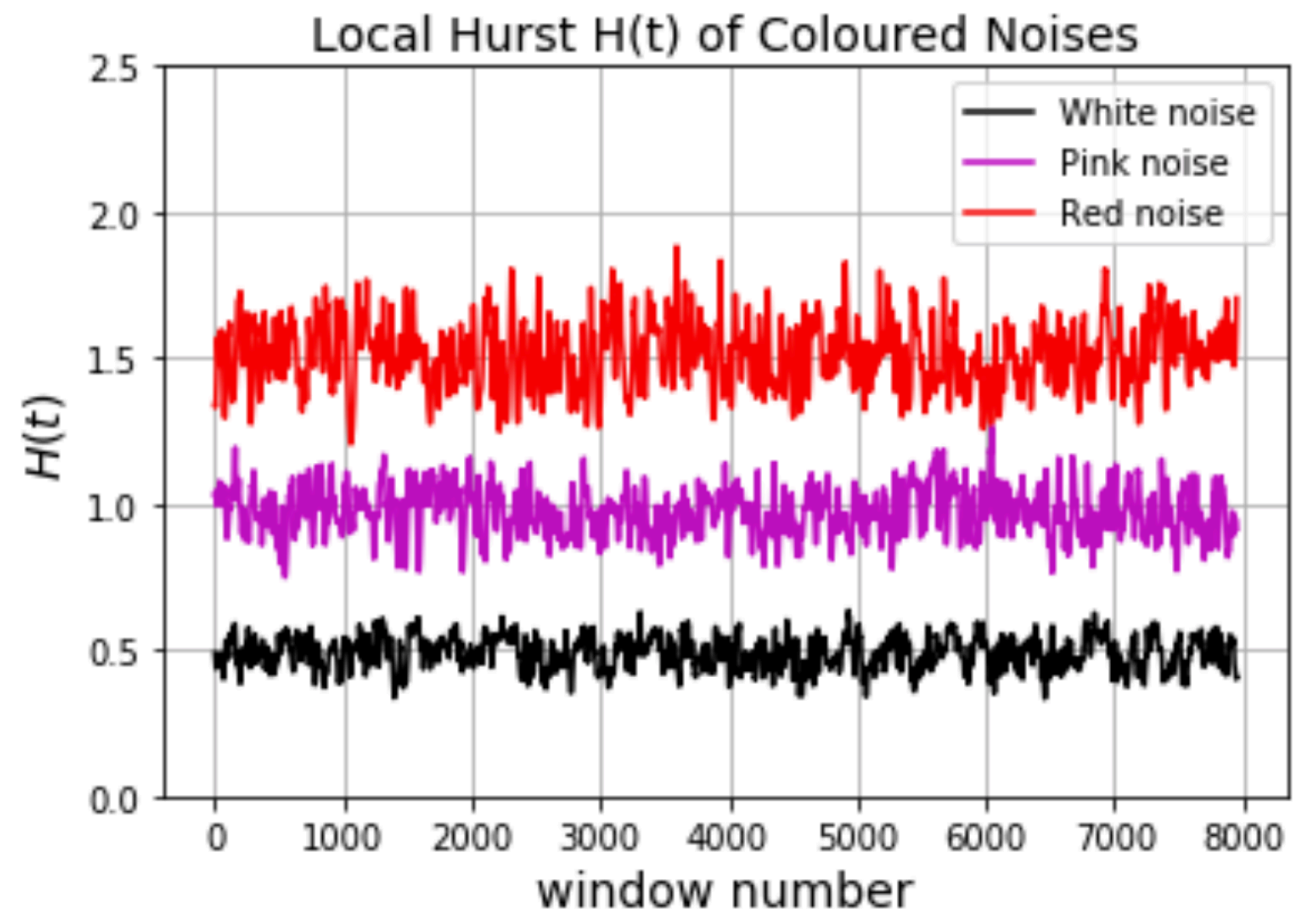
- Similar to DFA, but time series divided into small **overlapping** windows of length δ
- Small overlapping windows \rightarrow fluctuations quantified “locally” as a function of time
- RMS is computed in each small window length δ
- **$H(t)$ values:** slope of the straight lines connecting the RMS values, computed in each small window, to the value of $F(N)$
- Full details in \rightarrow [**LINK to Ihlen paper**](#)

Fathon Algorithm Testing: $H(t)$ for Coloured Noises



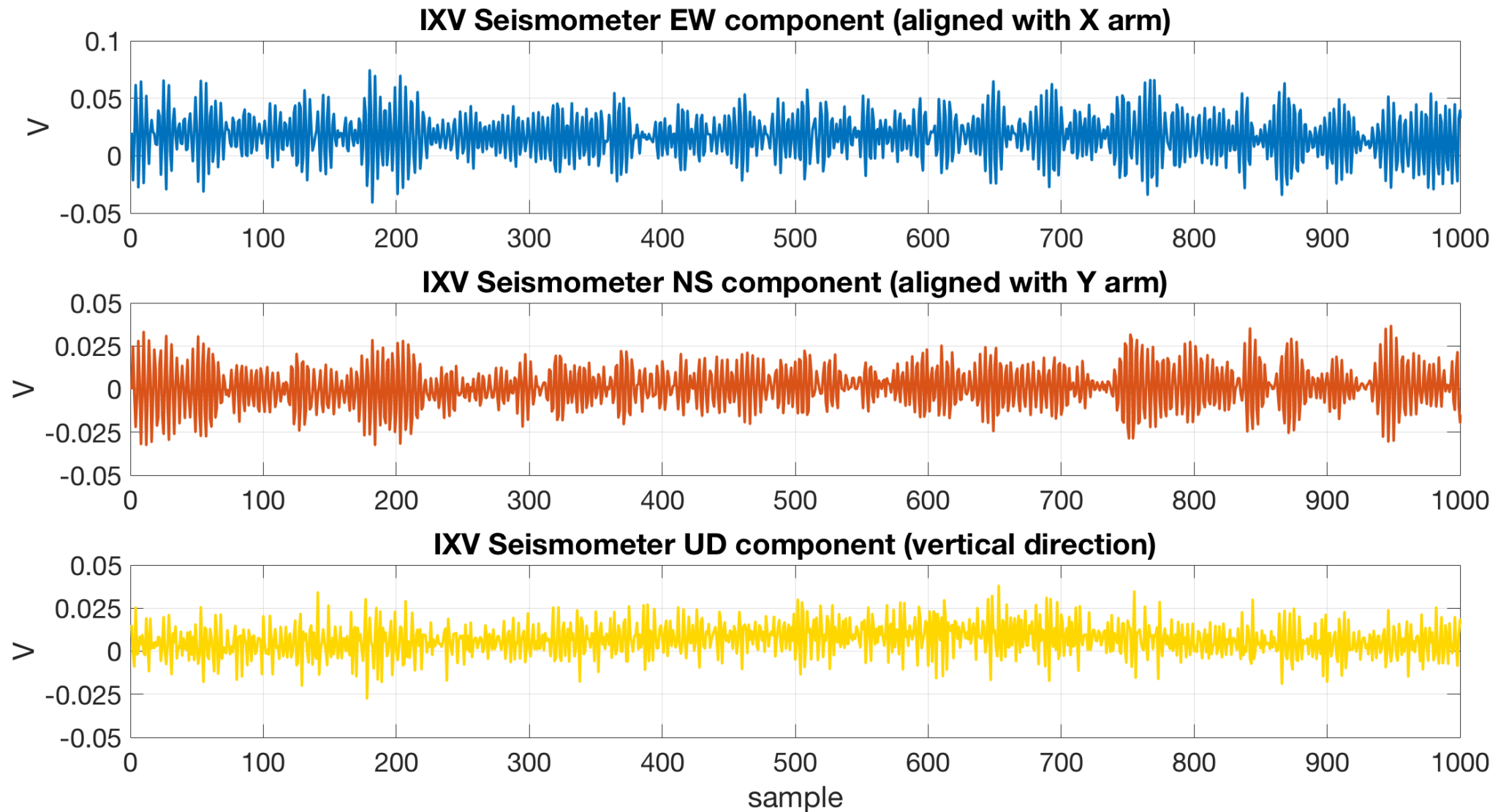
- **Red noise:** $H=1.5$
- **Pink noise:** $H=1$
- **White noise:** $H=0.5$

- $H(t)$ of coloured noises correctly estimated.
- Small windows of $\delta=50$ used for testing



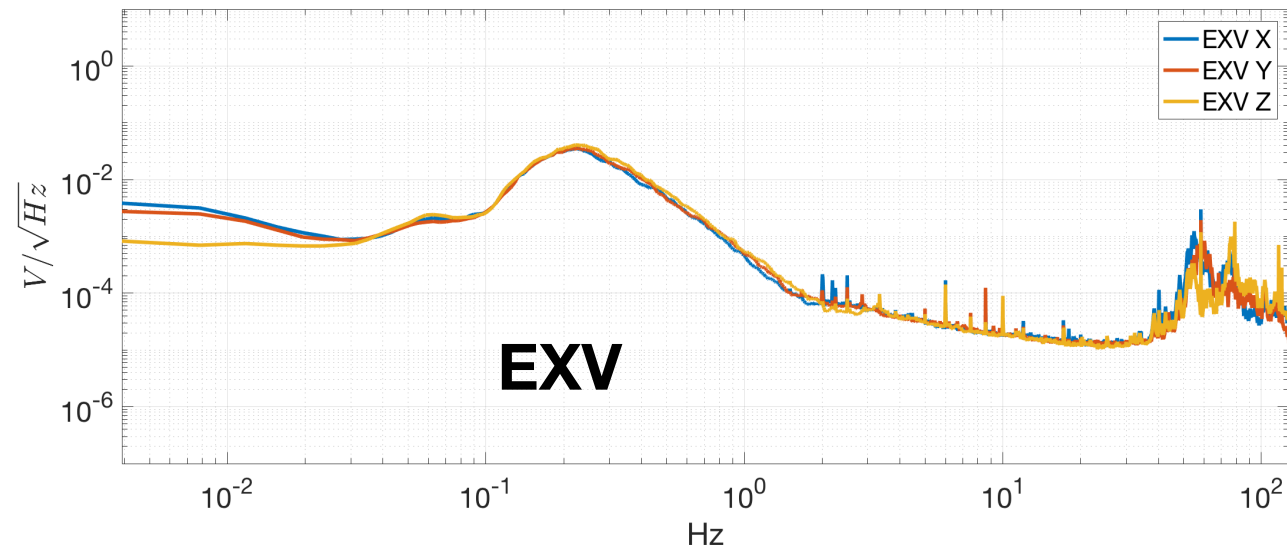
KAGRA Triaxial Seismometers Data Analysis

Persistency of KAGRA Seismometers

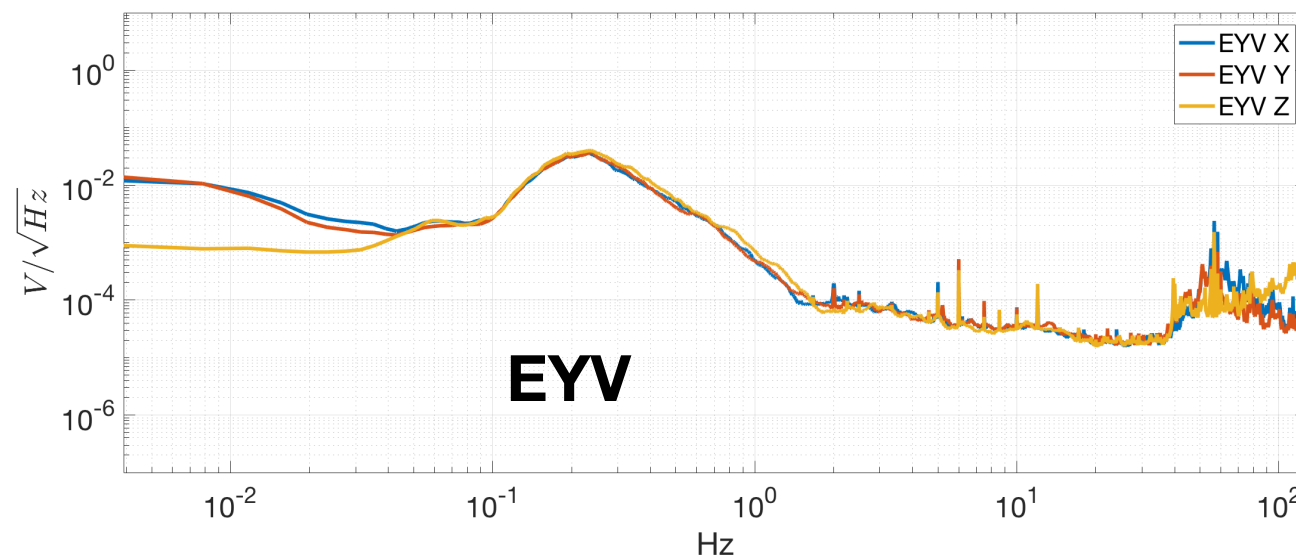


- **Example:** 4s from IXV seismometer, monitoring the corner area of KAGRA. $F_{\text{sampl}}=256\text{Hz}$
- Triaxial Seismometer: EW, NS (horizontal), UD (vertical)
- UD has lower persistency compared to EW and NS \rightarrow lower values of H

Average ASD of KAGRA seismometer data

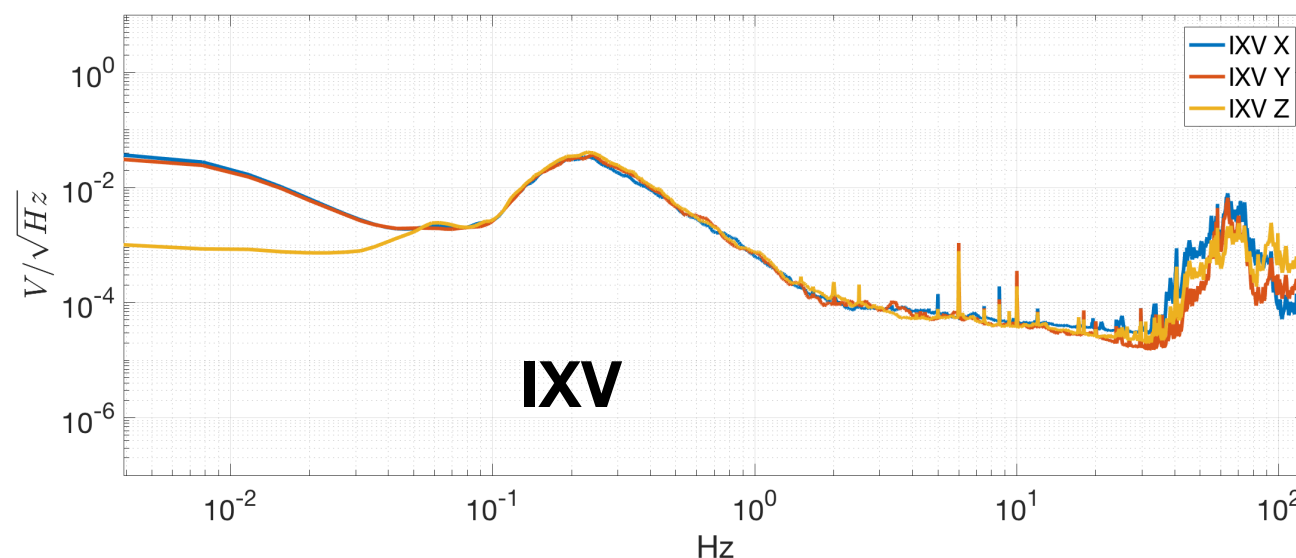


- Analysed data 31/7/2019 - 19/6/2020 (100 non consecutive hours)

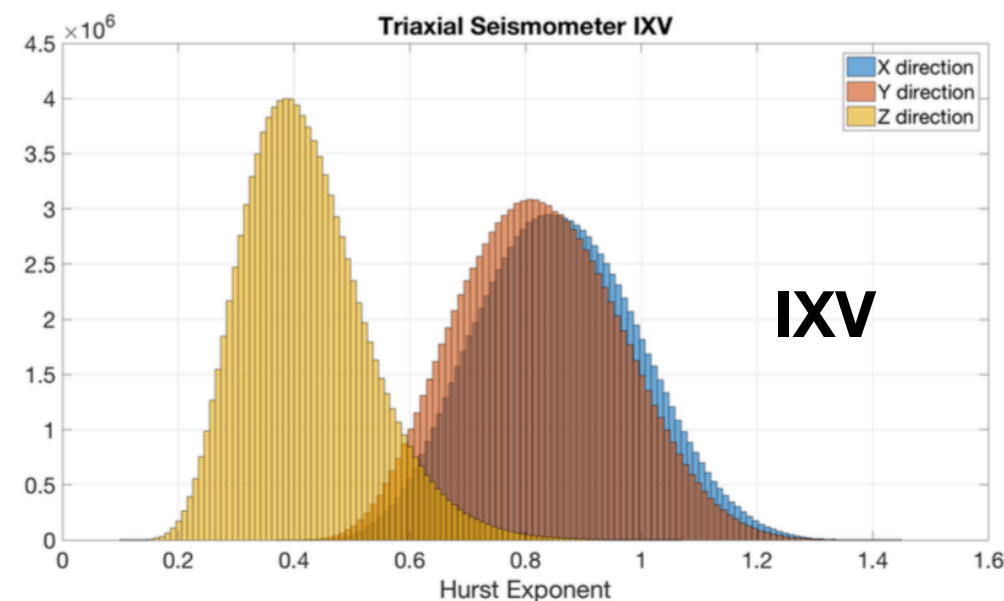
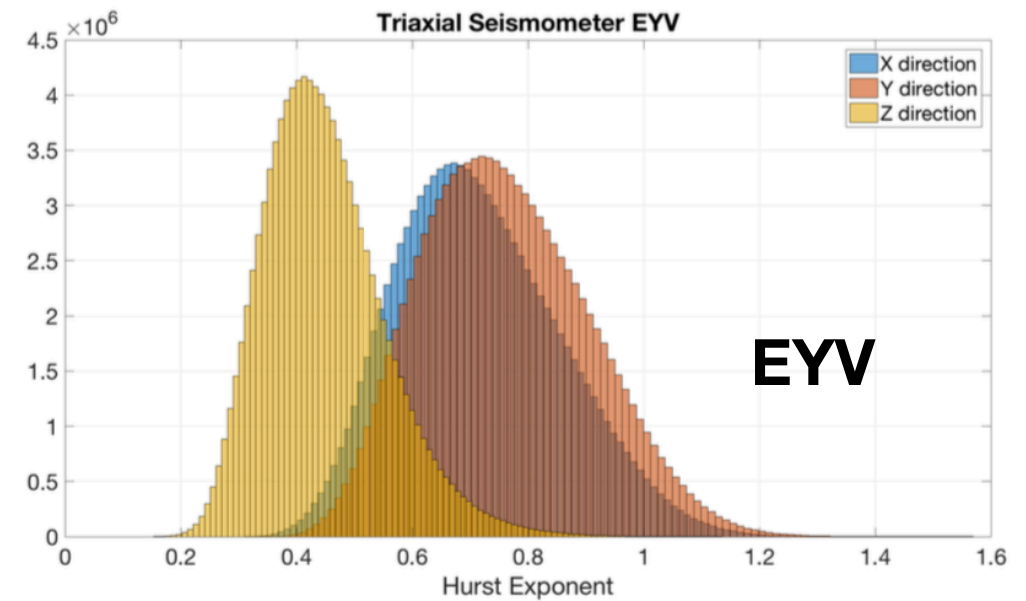
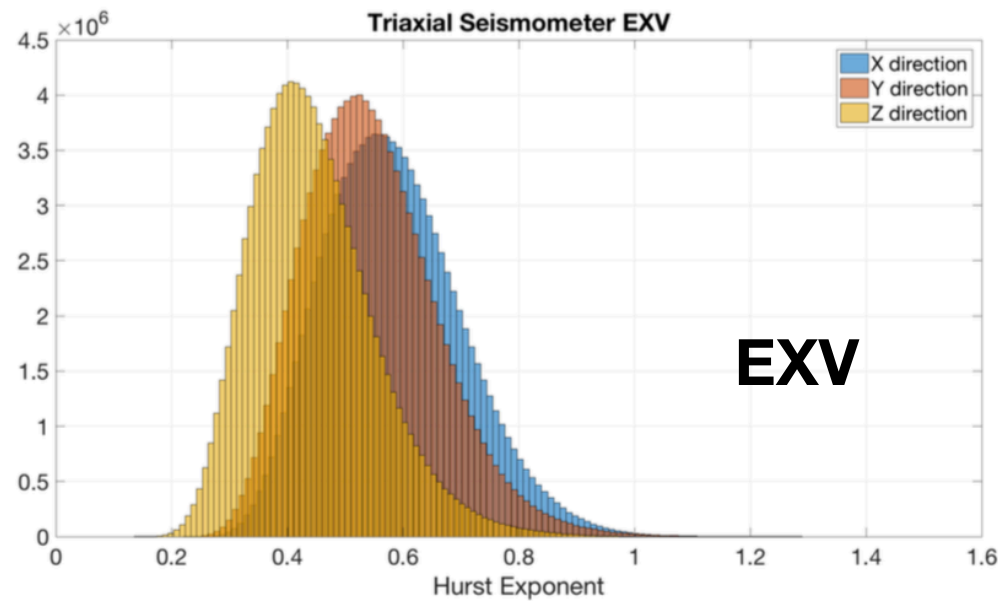


Average ASD

- Vertical component (yellow) -> Lower ASD in low frequency region for three seismometers
- EXV horizontal components (red blue): Lower ASD in the low frequency region compared to EYV and IXV



Histograms of Local Hurst Exponent $H(t)$



- Vertical components \rightarrow lower H for the three seismometers
- EXV horizontal components \rightarrow lower H compared to EYV and IXV
- Data analysis parameters: window length $\delta = 500$, linear detrend y^{fit} in each window

Conclusions

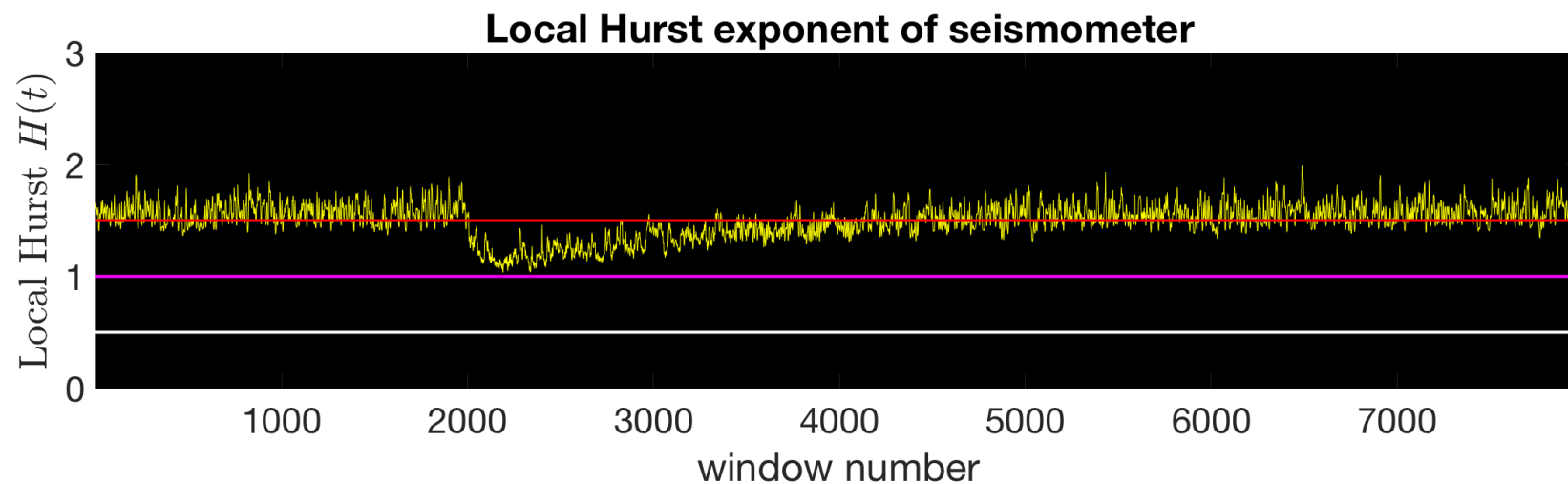
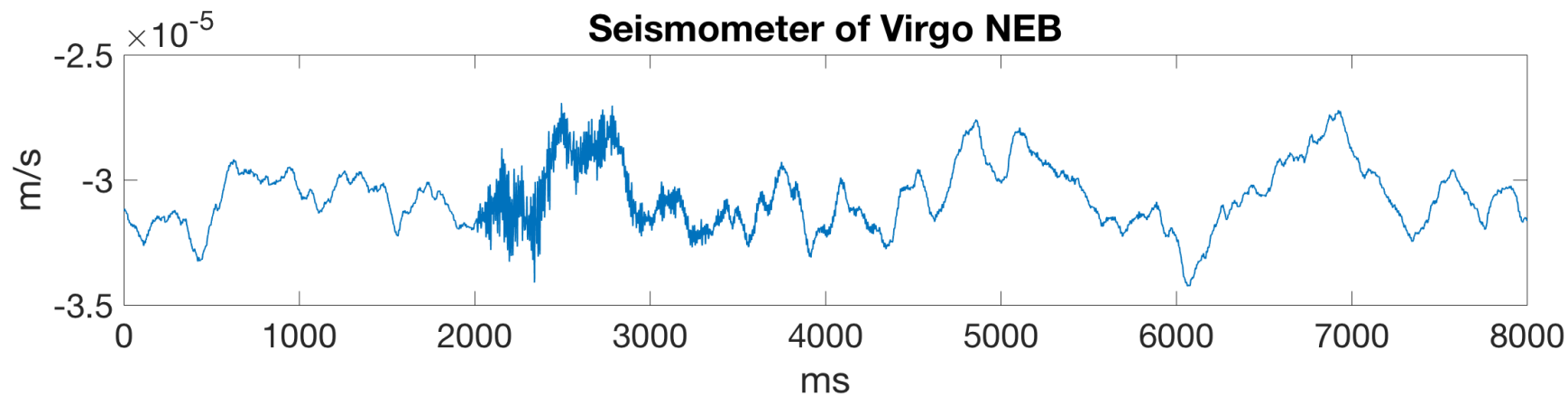
- The local Hurst exponent $H(t)$ was computed for three triaxial seismometers monitoring KAGRA
 - Vertical component of three seismometers has lower persistency \rightarrow lower H
 - Seismometer EXV (horizontal components) \rightarrow lower H compared to EYV and IXV seismometers
 - **Possible explanation:** reduced influence of human activity in X end area (no exit point), different from EYV and IXV
 - Water flow inside the mine where KAGRA is located? Further analysis needed, using other environmental noise data
-
- **$H(t)$ can monitor over time the persistency of interferometer's data. Useful to monitor stability?**
 - **Fathon** code available online: <https://pypi.org/project/fathon/>
 - Fathon Contacts: **alessandro.longo@uniroma3.it** **stefano.bianchi@uniroma3.it** **wolfango.plastino@uniroma3.it**

EXTRA SLIDES

Applications to Virgo Data

Local Hurst Exponent of Virgo Seismometer in NEB

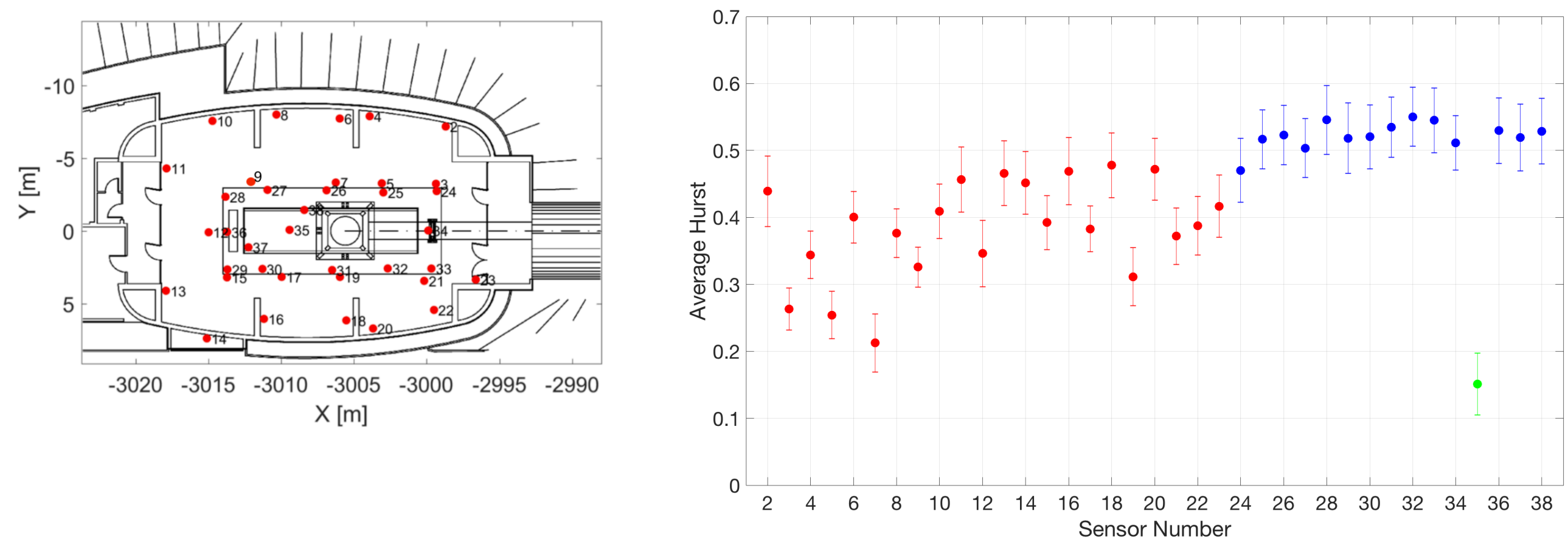
- Seismic noise in Virgo seismometer due to acoustic noise injection performed in North End Building -> change in data persistency can be monitored with high resolution using $H(t)$



- $H(t)$ tracks change of persistency in Virgo seismometer
- Small windows of $\delta=10$

Alessandro Longo, Stefano Bianchi, Wolfango Plastino et al. Adaptive denoising of acoustic noise injections performed at Virgo Interferometer. Pure and Applied Geophysics 177, 3395–3406, 2020.

Persistency of Seismometer Array in Virgo West End Building



Alessandro Longo, Stefano Bianchi, Wolfango Plastino, Bartosz Idzkowski, Maciej Suchinski and Tomasz Bulik, *Fractal analysis of data from seismometer array monitoring Virgo Interferometer*. Pure and Applied Geophysics 177, 2597-2603, 2020.

- **Left:** Coordinates of seismometer array **Right:** Average Hurst of seismometer array
- **Red:** Sensors not on concrete platform hosting Superattenuator tower.
- **Blue:** Sensors on the concrete platform
- Seismic noise in WEB has different persistency depending on sensor location