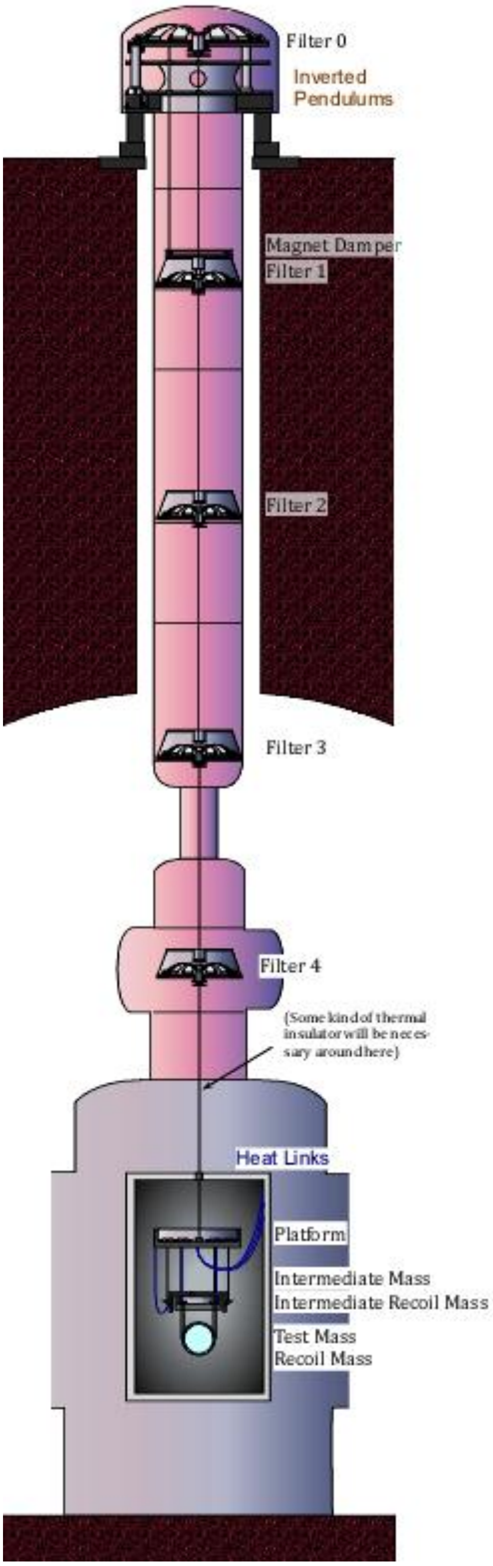
Angular Fluctuation of the test mass due to the seismic motions

July 13, 2011 Takanori Sekiguchi

Schematic View of the Type A Vibration Isolation System



F0: Filter 0

F1: Filter 1

F2: Filter 2

F3: Filter 3

F4: Filter 4

PF: Platform

IM: Intermediate Mass

IRM: Intermediate Recoil Mass

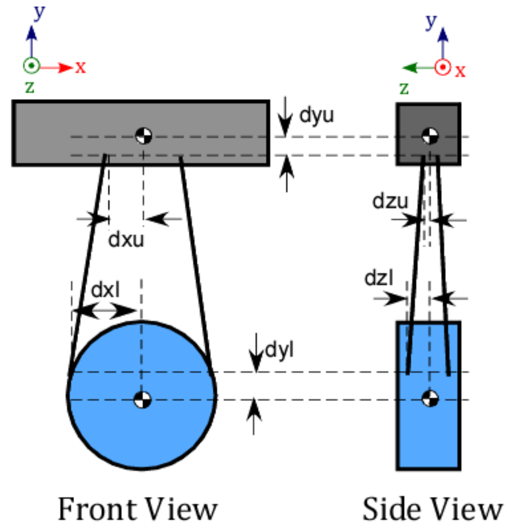
TM: Test Mass

RM: Recoil Mass

MD: Magnet Damper (Magnet Disc)

Although some kind of thermal insulator will be necessary between F4 and PF, it is not taken into account in the current simulation.

Parameter List Assumed in the Simulation



mass: mass of bodies [kg]

moix(moiy, moiz): moment of inertia around x(y,z)-axis [kg m^2]

fGAS: resonant frequencies of GAS filters [Hz]

nw: number of wires

matw: material of wires

lNw: natural length of wires [m]

dw: wire diameter [mm]

dyu(dyl): break-off between suspension points and CoM [mm]

dxu(dxl): x-separation of wires [mm]

dzu(dzl): z-separation of wires [mm]

	F0	F1	F2	F3	F4	PF	IM	IRM	TM	RM	MD
mass	574	104	90	87	60	60	60	60	30	30	30
moix	15.00	4.00	4.00	4.00	4.00	0.97	0.80	0.96	0.17	0.45	0.92
moiy	20.00	6.44	6.44	6.44	6.00	1.94	1.88	2.35	0.17	0.45	1.83
moiz	15.00	4.00	4.00	4.00	4.00	0.97	1.22	1.51	0.23	0.57	0.92
fGAS	-	0.33	0.33	0.33	0.33	0.33	3.00	3.00	-	-	-
nw	-	1	1	1	1	1	4	4	4	4	3
matw*	-	MS	MS	MS	MS	MS	W	W	SA	W	C70
lNw	-	2.100	2.100	2.350	2.085	2.084	0.500	0.502	0.300	0.300	1.925
dw	-	3.20	3.00	2.80	2.50	2.10	0.60	0.60	1.60	0.72	0.30
dyu	-	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	70.0
dyl	-	10.0	10.0	10.0	10.0	-5.0	1.0	1.0	1.0	1.0	1.0
dxu	-	-	-	-	-	-	125	220	125	145	350
dxl	-	-	-	-	-	-	125	220	125	145	
dzu	-	-	-	-	-	-	75	166	15	15	
dzl	-	-	-	-	-	-	75	166	15	15	

* MS: Maraging steel, SA: Sapphire, W: Tungsten, C70: C70-steel

These parameters are based on the document "Type-A filter chain parameter set" (T1100401-v1) and the design of the cryogenic payload that was drawn by R. Takahashi in Oct. 2010.

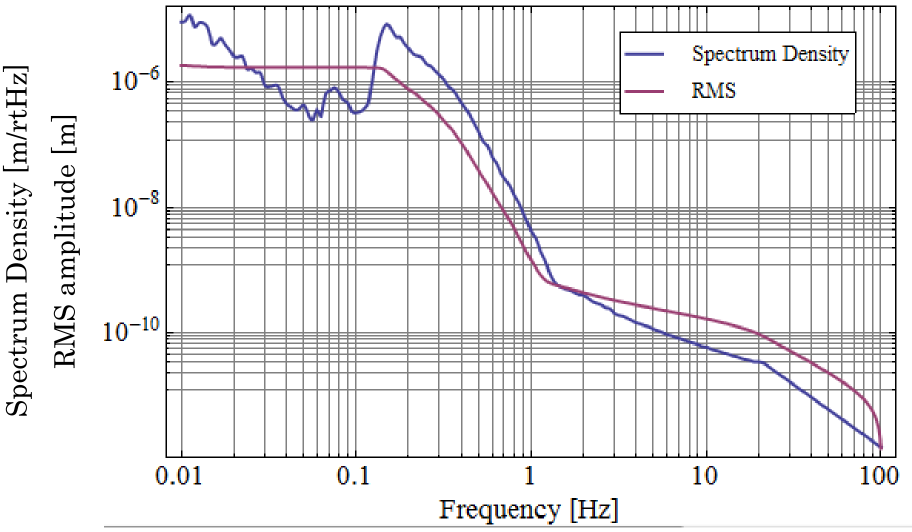
Other Assumptions

* Inverted pendulums are tuned at 30 mHz,

* All the standard GAS filters are tuned at 330 mHz and the mini-GAS filters are tuned at 3.0 Hz.

* In the simulation, the heat link works as an ideal spring with resonant frequency of 50 mHz when it is attached to 120 kg mass (this assumption is based on Aso-san's FEM analysis). We assume that the heat link is attached to the CoM of the platform.

Seismic Noise Level in Kamioka



Graph 1: Seismic noise level in the Kamioka mine

* This seismic noise spectrum was measured by Araya-san in 2007.

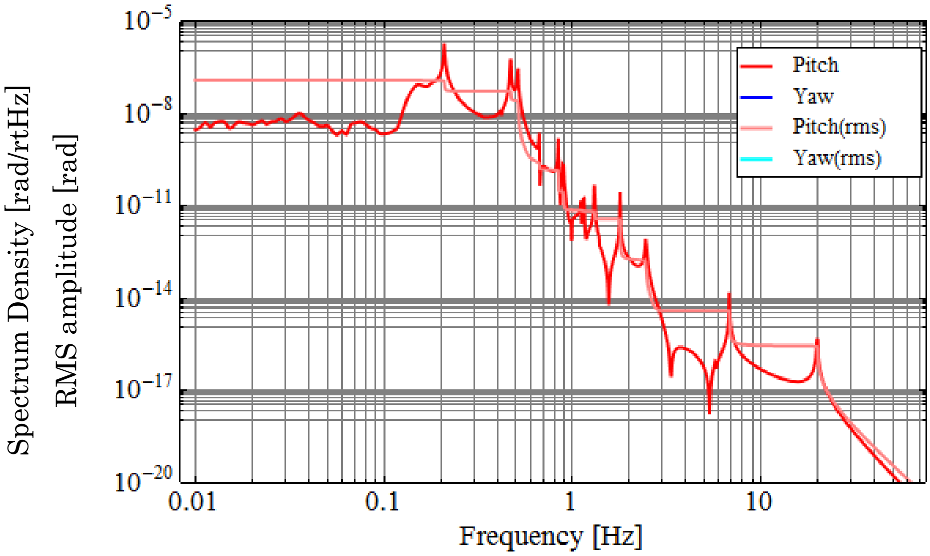
* <20 Hz: measured values, >20Hz: interpolation

* These data are measured in a very noisy day (in a very bad seismic condition). Normally, the rms of the ground seismic motion in Kamioka is several 10^{-7} m, but in a noisy day, the rms reaches several 10^{-6} m.

* We have to operate the interferometer even in noisy days, so we use this noisy plot in the simulation.

Angular Fluctuation of the Test Mass

If there is no asymmetry in wire length, suspension points, or etc., the angular fluctuation of the mirror will be relatively small. The following plot shows the angular displacement of the test mass due to the seismic motion without any asymmetry (and without any damping).



Graph 2: Angular fluctuation of the TM without any asymmetry

In the simulation, transfer functions from the ground x(y,z) motion to pitch(yaw) motion of the test mass are calculated. The plot shown above is a sum of the couplings from x, y and z ground motion.

If there is no asymmetry in the system, yaw modes are never coupled with other DoFs, so the amplitude of the yaw motion of the test mass is zero. (Of course, there might be a twist (or tilt) of the ground, so the yaw motion of theTM will never be zero actually.) On the other hand, pitch modes are always coupled with longitudinal translation modes, because of the mismatch of the vertical positions of the suspension points and CoM (see Figure 1). So, the pitch motions due to this coupling can be seen in the plot. The rms amplitude of the yaw motion is estimated as several 10^{-7} rad.

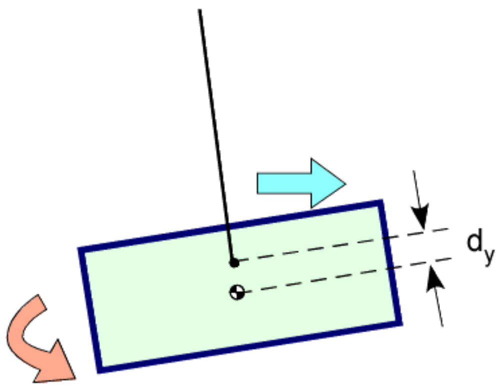


Figure 1: Translation-tilt coupling

However, there is no symmetric mechanical system, and the angular motions of the bodies are quite sensitive to the asymmetry. In order to estimate the angular fluctuation of the mirrors, we need to consider the asymmetry of the system.

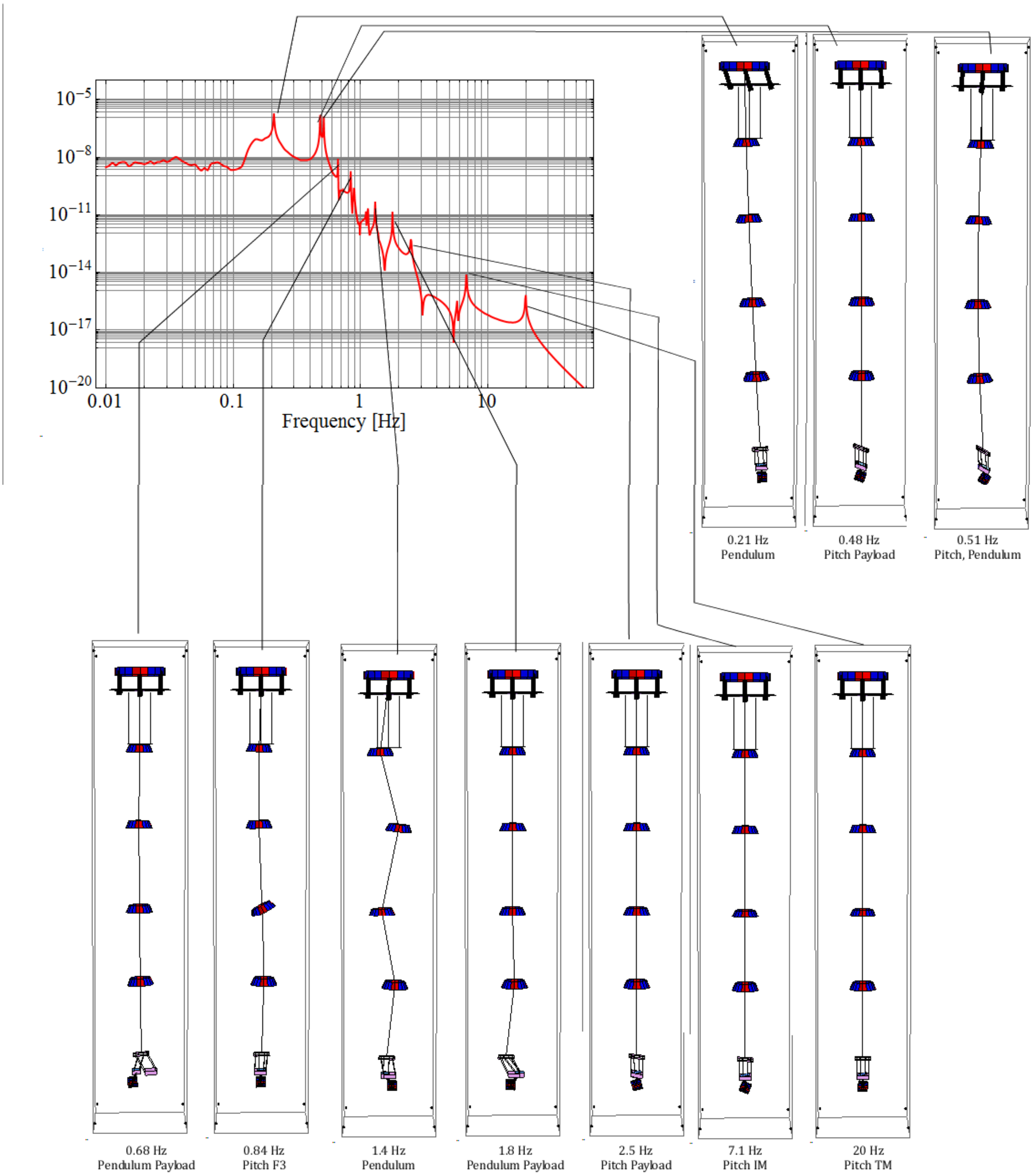


Figure 2: Modal analysis of the longitudinal-pitch modes

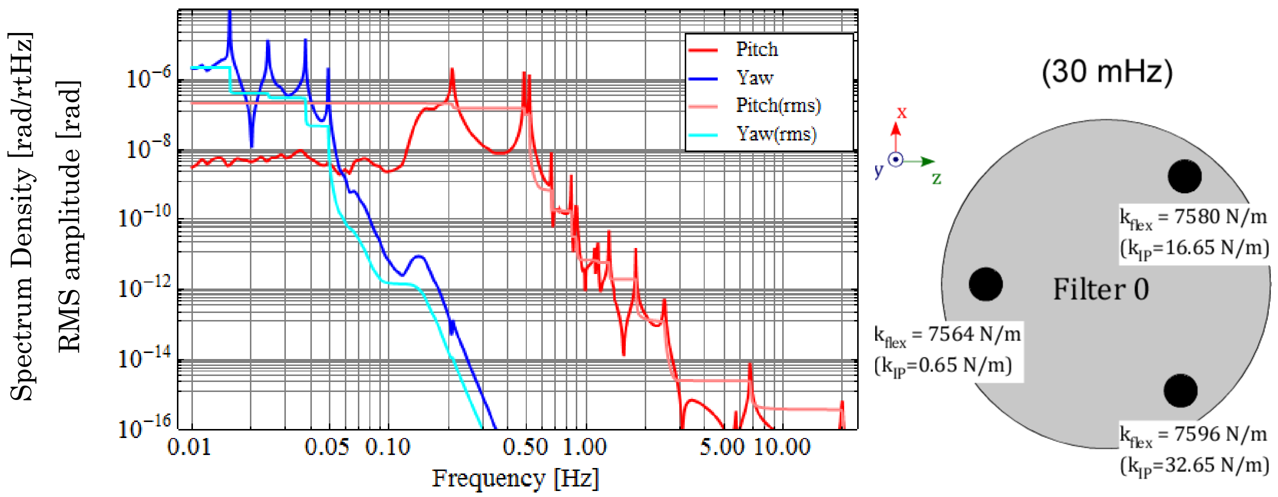
*** Asymmetry in the Spring Constants of the Inverted Pendulums**

The stiffness of an inverted pendulum depends on the thickness of the flex joint. If we assume circular cross section for the flex joint, the spring constant of the joint is proportional to the forth power of the thickness of the flex joint. It means that the 0.1% error on the thickness is amplified ~0.4% in the stiffness. In addition, the spring constant of the inverted pendulum is reduced by the weight on the top of the leg,

$$k_{IP} = k_{flex} - Mg / L$$

So, for example, if the stiffness of the flex joint is k_{flex}=2000 N/m and it was reduced by Mg/L=1950 N/m, the effective stiffness of the inverted pendulum will be k_{IP}=50 N/m. If we assume 1% error in the stiffness (k_{flex} =1980 N/m), the effective stiffness of the IP becomes k_{IP}=30 N/m. This is about half of the former IP. This indicates that small errors in the stiffness are amplified by a large factor. This problem will be more critical when the pre-isolator stage is tuned at a lower frequency.

When there is asymmetry in the stiffness of the IPs, the translational motion of the F0 is coupled with the yaw motion. Graph 3 shows the angular fluctuation of the TM, assuming ±0.2% errors in the stiffness of the flex joints.



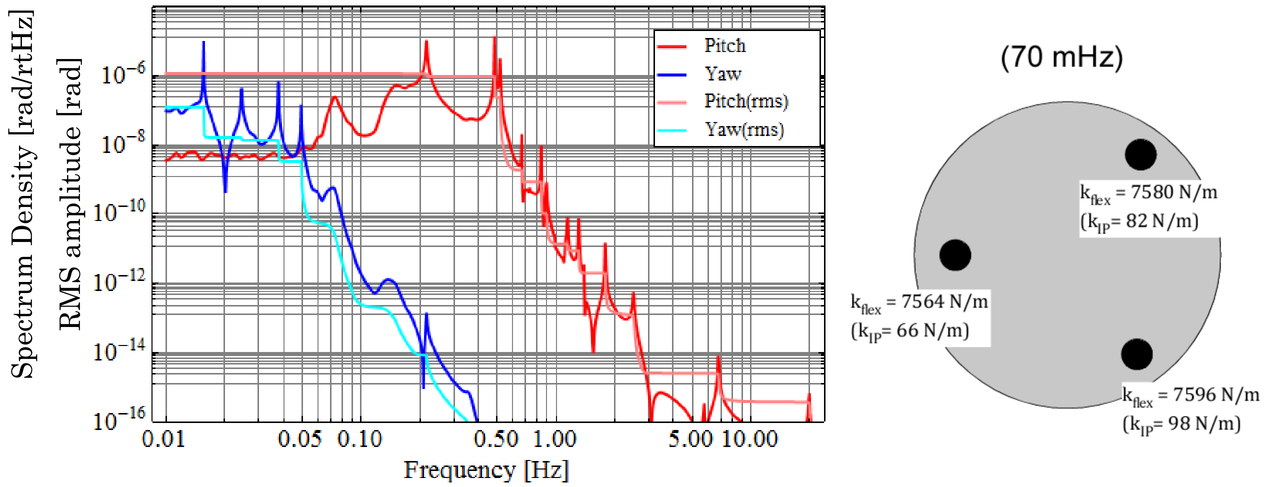
Graph 3: Angular fluctuation of the TM with asymmetry in the stiffness of the IPs (30 mHz)

In this case, the yaw motion of the TM is excited in very low frequencies (the pitch mode is not affected). The large peaks at 0.01~0.05Hz are the torsion modes of the maraging wires. The amplitude of the yaw motion is dramatically cut off in high frequencies due to these torsion springs. So the yaw motion in high frequencies is not a problem, while the rms amplitude of this mode reaches more than 10^{-6} rad.

To reduce this large yaw motion, we need 1) to damp the torsion modes of the wire, and 2) to make the asymmetry in the stiffness of the IPs much smaller. Former one is achieved by applying eddy current damping at the top of the SAS chain (this will be discussed later). To achieve 2), we may have to balance the weight distribution to the IP legs. If it is difficult, we can reduce the asymmetry of the stiffness by tuning the resonant frequency of the pre-isolator stage higher.

Graph 4 shows the angular fluctuation of the mirror in the case that the resonant frequency of the pre-isolator stage is tuned at 70 mHz, with the same asymmetry in the stiffness of the flex joints. To make the resonant frequency higher, the mass of the F0 is reduced by 10 kg. In this case, the rms amplitude of the TM yaw motion is reduced to 10^{-7} rad.

On the other hand, the rms amplitude of the pitch mode gets larger. This is because the loss angle of the pre-isolator stage gets smaller, and the resonance peaks around 0.2-1.0 Hz get larger. These peaks can be also damped by the eddy current damping in the top stage, so we don't need to worry about them.



Graph 4: Angular fluctuation of the TM with asymmetry in the stiffness of the IPs (70 mHz)

As a conclusion,
* Asymmetry in the stiffness of the flex joints may excite the yaw modes in low frequencies.
* In the worst case, rms amplitude of the TM yaw motion reaches several 10^{-6} rad.
* It is crucial to damp the torsion modes and make the asymmetry as small as possible.
* This asymmetry does not affect the pitch motion of the TM.

*** Misalignment of Suspension Points**

To attenuate the seismic noise in 6 DoFs, each GAS filter is suspended by a single wire. In the single wire suspension, there might be a misalignment of the suspension points as shown in the figure below.

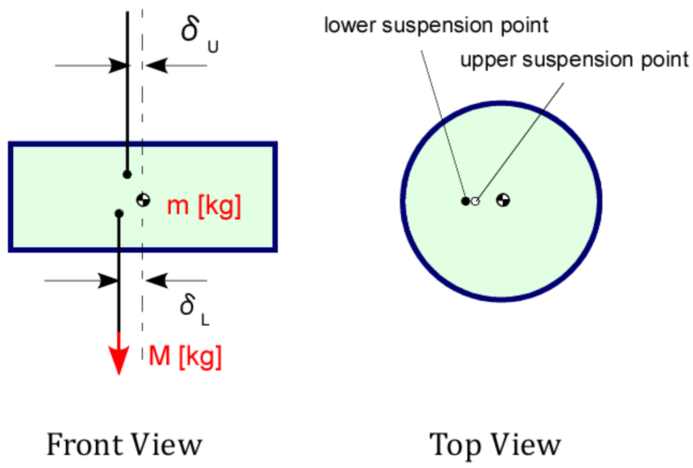


Figure 3: Misalignment of suspension points

In order to balance the filter, the following equation is required for the δ_L and δ_U.

$$(M + m)\delta_U = M\delta_L$$

If such a misalignment exists, horizontal-yaw and vertical-pitch couplings are expected.

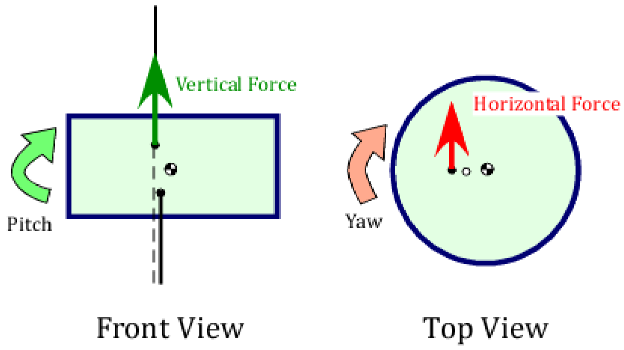
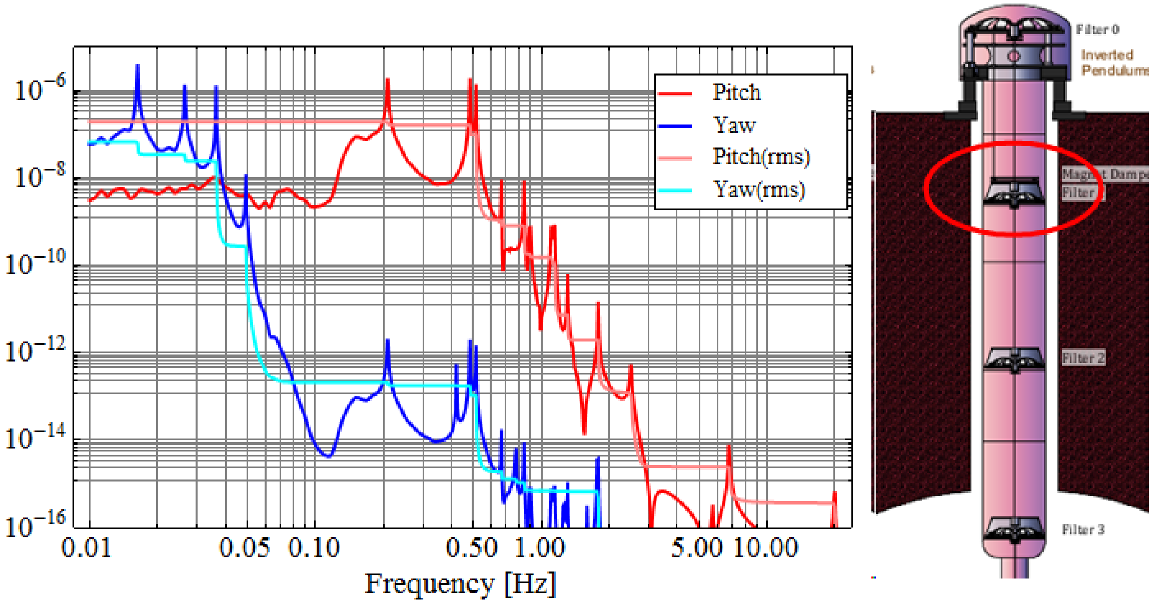
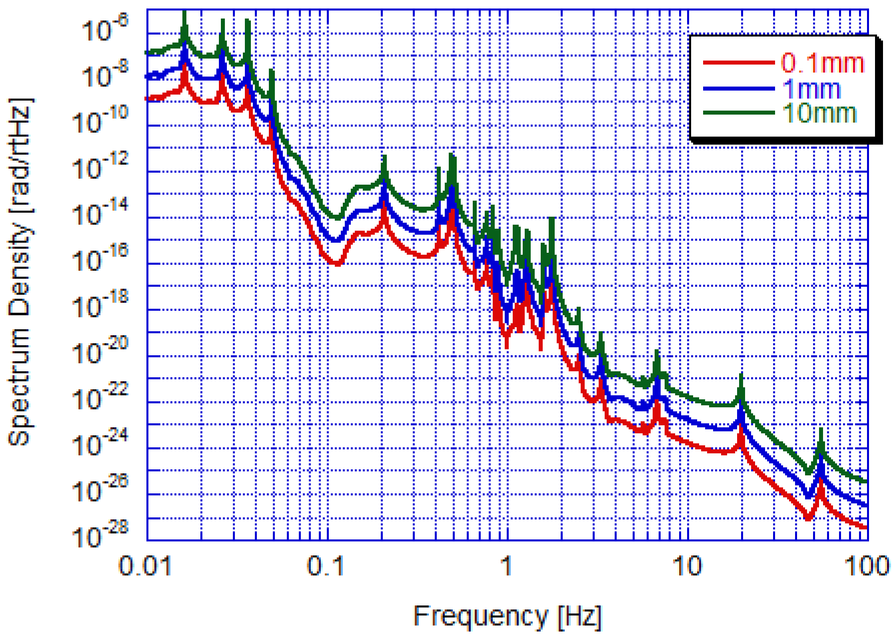


Figure 4: Couplings due to misalignment in suspension points

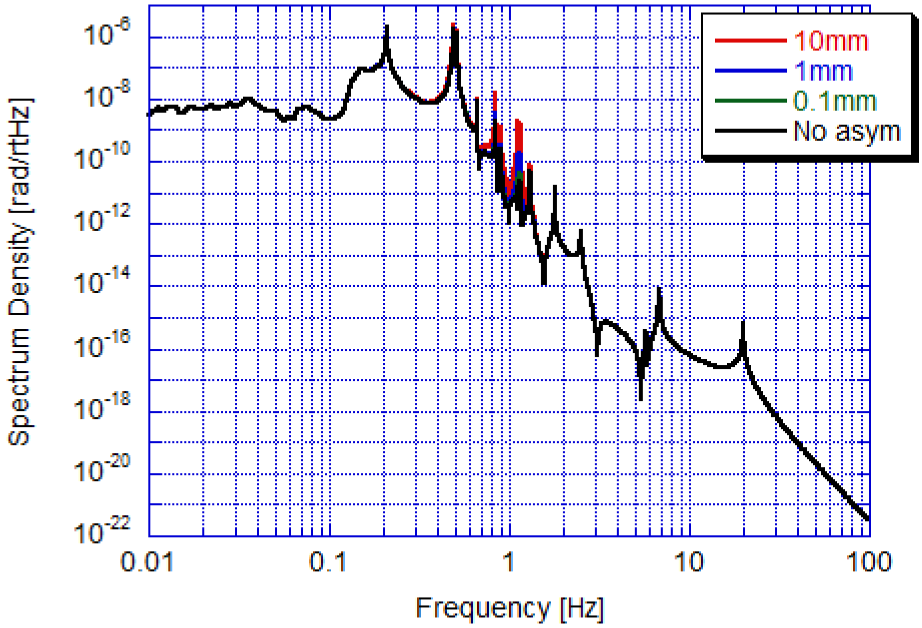
Graph 5 shows the angular fluctuation of the mirror when there is a misalignment in the F1 (δ_L =5 mm in z-direction). As expected, the yaw motion of the TM is excited due to the transversal-yaw coupling. When the misalignment gets larger, the yaw motion of the TM also gets larger (see Graph 6). On the other hand, the pitch motion of the TM is not largely affected by this misalignment (see Graph 7). This is because the longitudinal-pitch mode coupling is relatively larger than the vertical-pitch mode coupling due to the misalignment. Only on the region around 1 Hz, the pitch motion is affected by the misalignment. Figure 5 shows the resonant mode analysis about the yaw plot.



Graph 5: Angular fluctuation of the TM with misalignment of suspension points at the F1



Graph 6: Yaw fluctuation of the TM, changing the value of δ_L



Graph 7: Pitch fluctuation of the TM, changing the value of δ_L

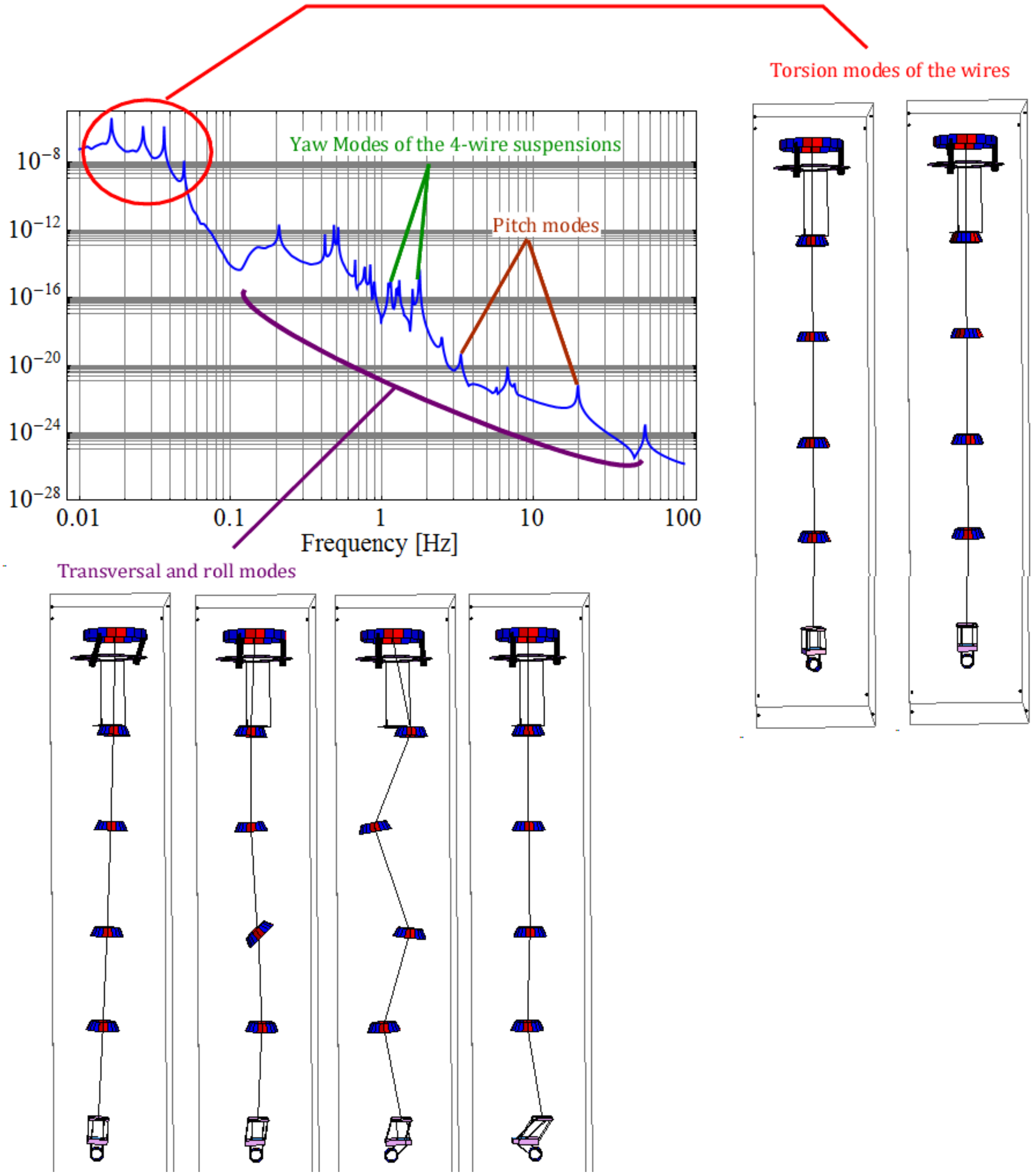
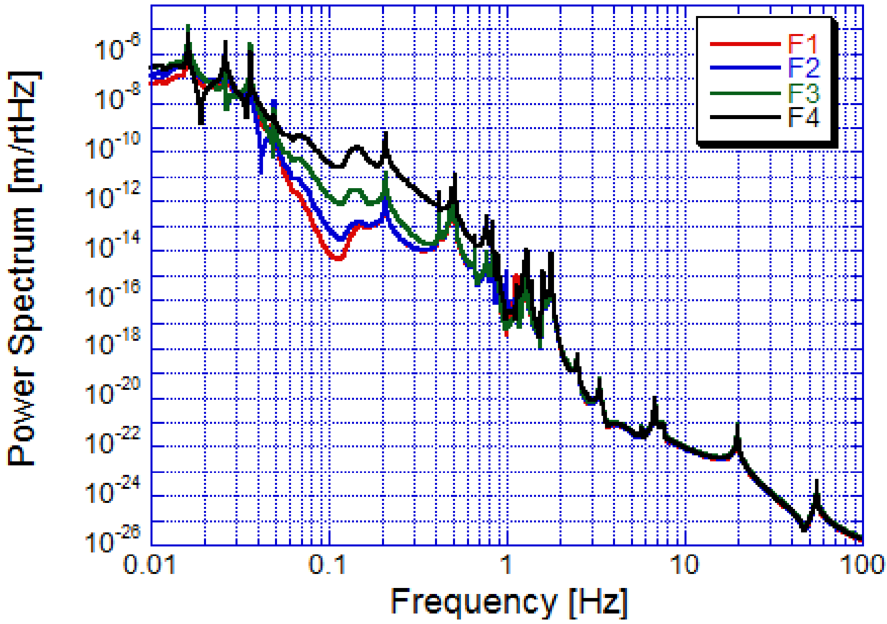


Figure 5: Modal analysis about the yaw plot

Graph 8 shows the yaw motion of the TM with the same misalignment (δ_L =5 mm in z-direction) in the F1, F2, F3 and F4. The effect of the misalignment is more critical when it exists in a lower filter, because the yaw motion excited in an upper filter is cut off by the torsion pendulums, but the motion excited in a lower filter transmits directly to the TM. However, the yaw motions in higher frequencies (> 2 Hz) are almost the same in all cases. I do not figure out why they are corresponding, but it seems that the yaw motions in higher frequencies are coupled with not only transversal modes, but also longitudinal modes (in Figure 5, you can see the pitch-yaw couplings in high frequencies).



Graph 8: Yaw fluctuation of the TM with misalignment at the F1~F4

Graph 9 shows the pitch motion of the TM with misalignment in F1~F4. In contrast with yaw modes, pitch motions of the TM are not so largely affected by the position of the misalignment. Only the difference is the position of the peaks around 1 Hz, and those peaks are corresponding to the resonance mode about the pitch motion of each filter.

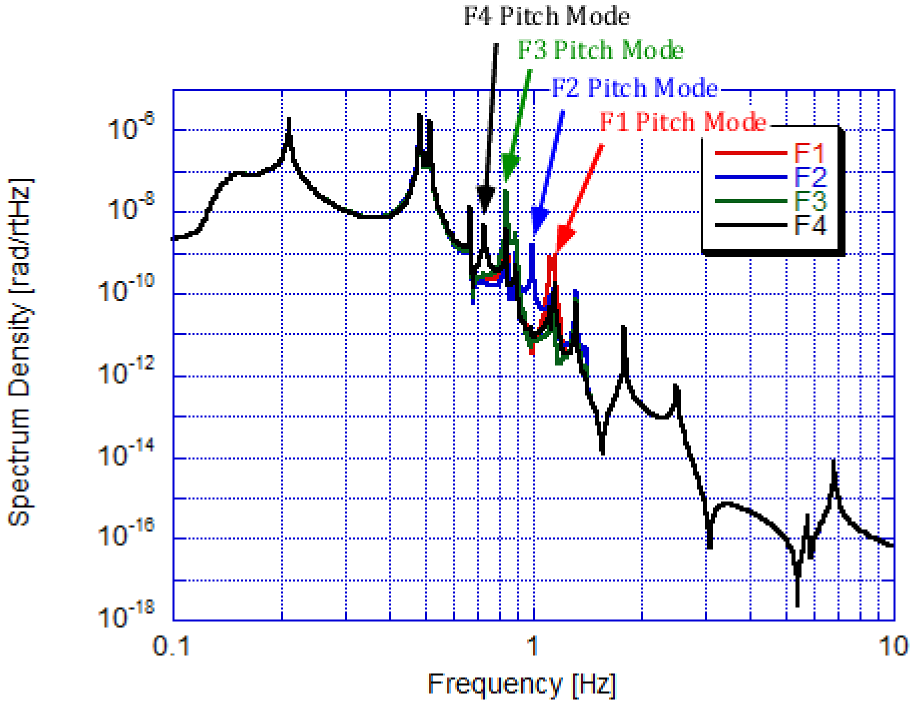
As a conclusion,

* Misalignment of the suspension points allows the horizontal-yaw couplings.

* The rms of the TM yaw motion will be 10^{-7} rad if we assume 1 cm misalignment. The amplitude is proportional to the distance between suspension points and CoM (δ_L).

* The misalignment in a lower filter (nearer to the TM) is more critical.

* This misalignment does not affect the TM pitch mode so largely.



Graph 9: Pitch fluctuation of the TM with misalignment of suspension points at the F1~F4

*** Asymmetry in 4-Wire Suspension**

Several kinds of asymmetry can be considered in a 4-wire suspension: asymmetry in 1) geometry of the suspension points, 2) wire length, and 3) thickness of the wires.

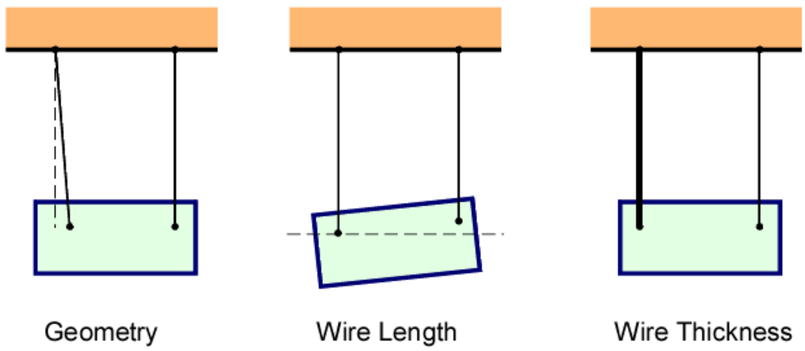


Figure 6: Asymmetry in a 4-wire suspension

(1) Geometry of the suspension points

If there is asymmetry in geometry, horizontal, vertical modes will couple with the pitch, roll, yaw modes in a complicated way. Although there are a lot of kinds of geometric asymmetry, firstly let me consider the following asymmetry shown in the Figure 7.

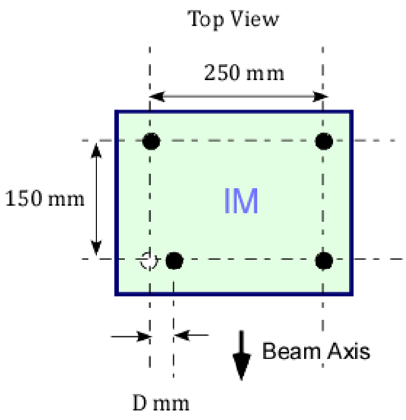
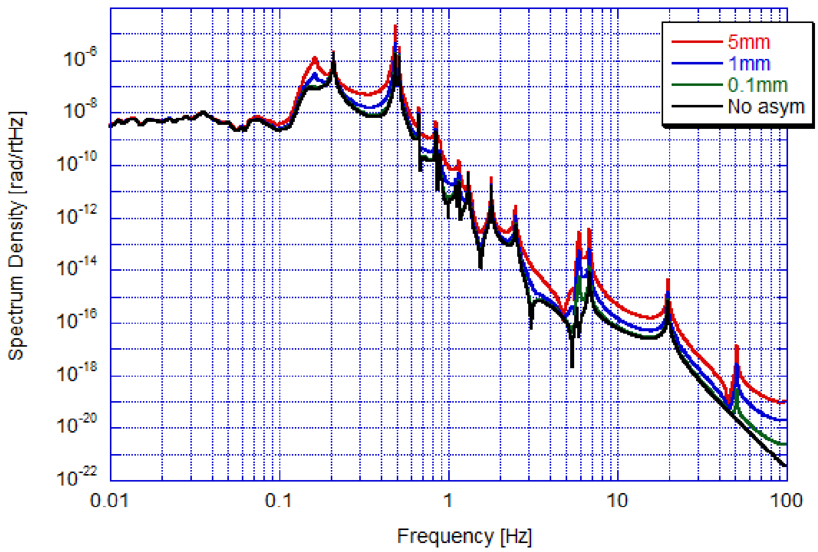


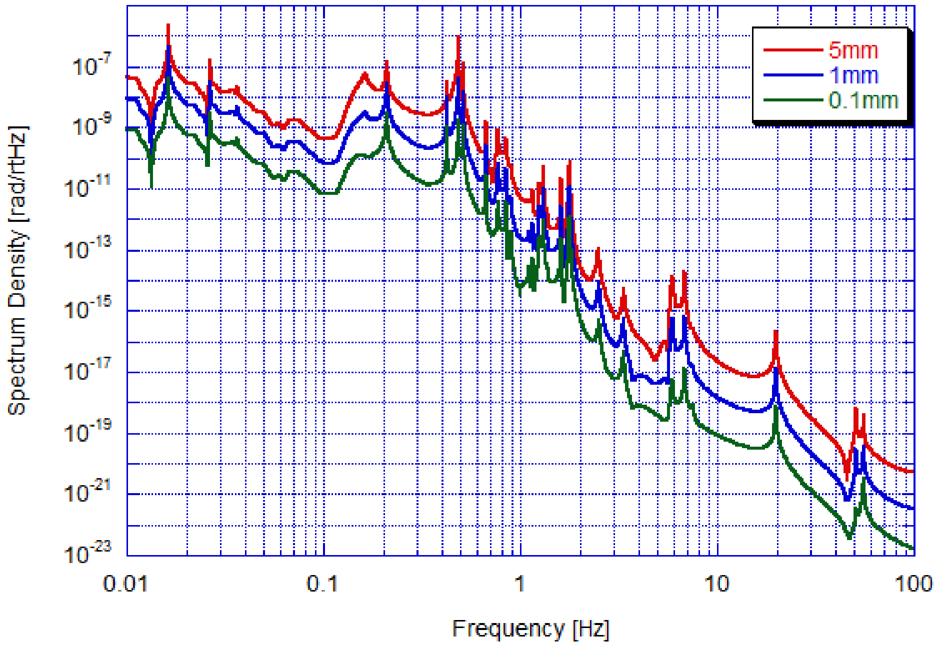
Figure 7: Geometric asymmetry in suspension points

Graph 10 and 11 show the TM angular mode amplitude with geometric asymmetry in the IM suspension. In this case, not only the yaw modes, but also pitch modes are affected. The main contributor to the pitch mode amplitude is the vertical-pitch coupling in the IM suspension. The broad peak at 0.16 Hz is the vertical bounce mode of the GAS filters (see Figure 8).

The yaw motion in low frequencies (<0.1 Hz) comes from the transversal-yaw mode coupling in the IM suspension. In high frequencies (>0.1 Hz), the yaw motion of the TM comes from various couplings, so it has a lot of peaks and constructs a complicated structure.



Graph 10: Pitch fluctuation of the TM with geometric asymmetry in the IM suspension



Graph11: Yaw fluctuation of the TM with geometric asymmetry in the IM suspension

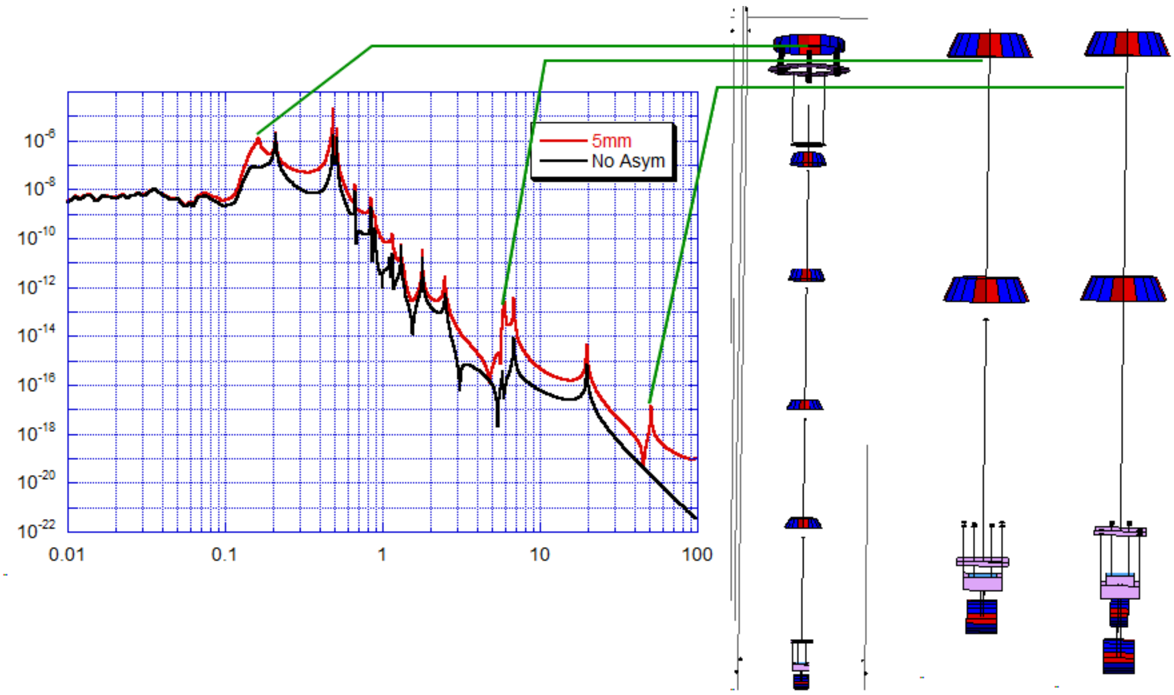
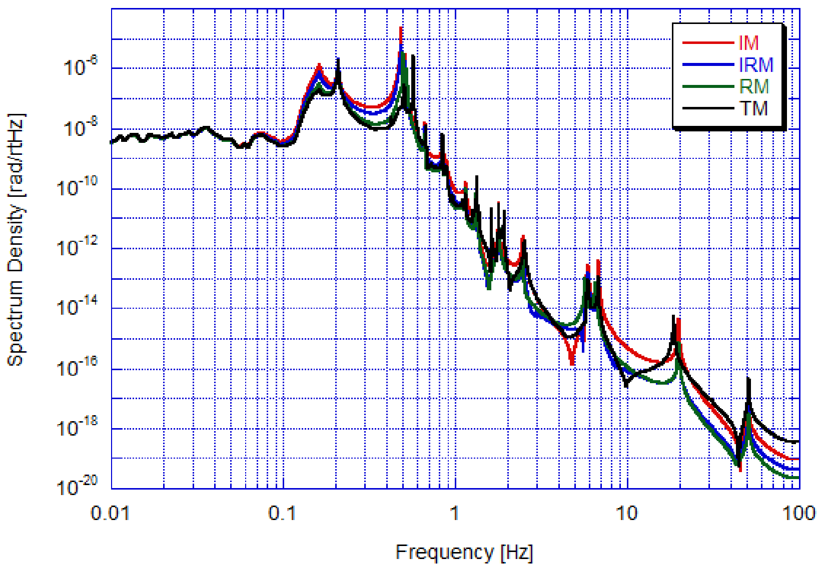


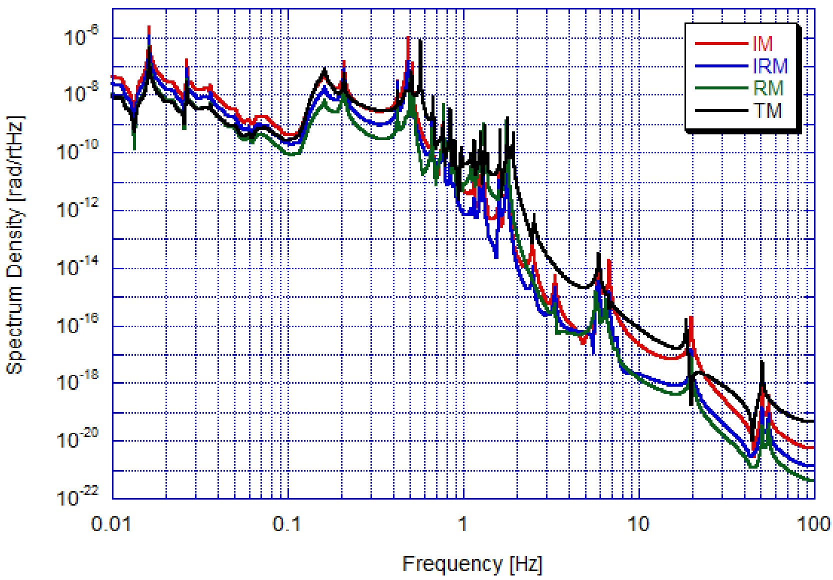
Figure 8: Modal analysis about the pitch plot

Graph 12 and 13 show the angular motion of the TM with the same (5 mm) geometric asymmetry in IM, IRM, RM and TM. In all cases, the plots show similar tendency. Asymmetry in the TM seems the most harmful for the yaw motions in high frequencies, while the amplitude of the yaw motion in the TM case is relatively small than other cases in low frequencies.

In any case, if we assume 5 mm asymmetry in suspension points, the rms amplitude of the TM angular motion becomes several 10^{-6}~10^{-7} rad for pitch, and 10^{-8} rad for yaw.



Graph12: Pitch motion of the TM with geometric asymmetry in IM, IRM, RM and TM

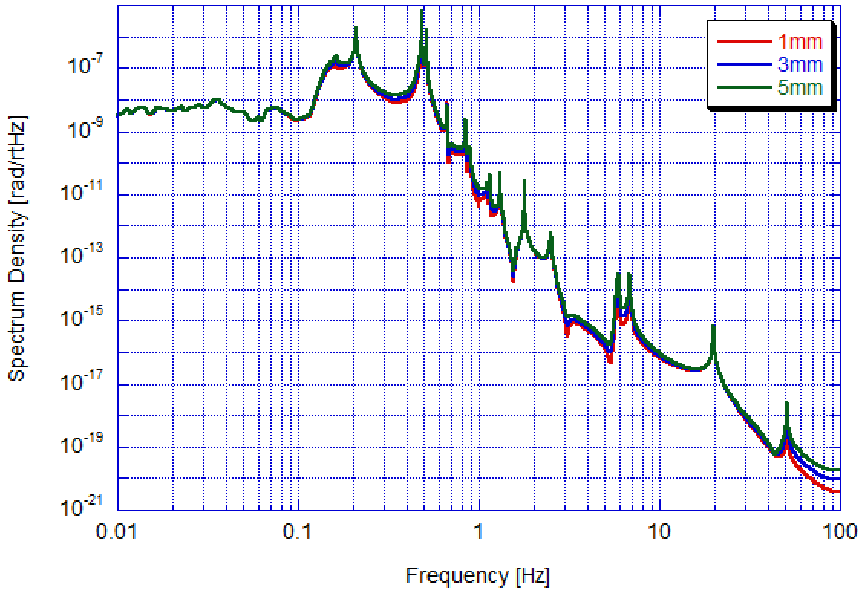


Graph13: Yaw motion of the TM with geometric asymmetry in IM, IRM, RM and TM

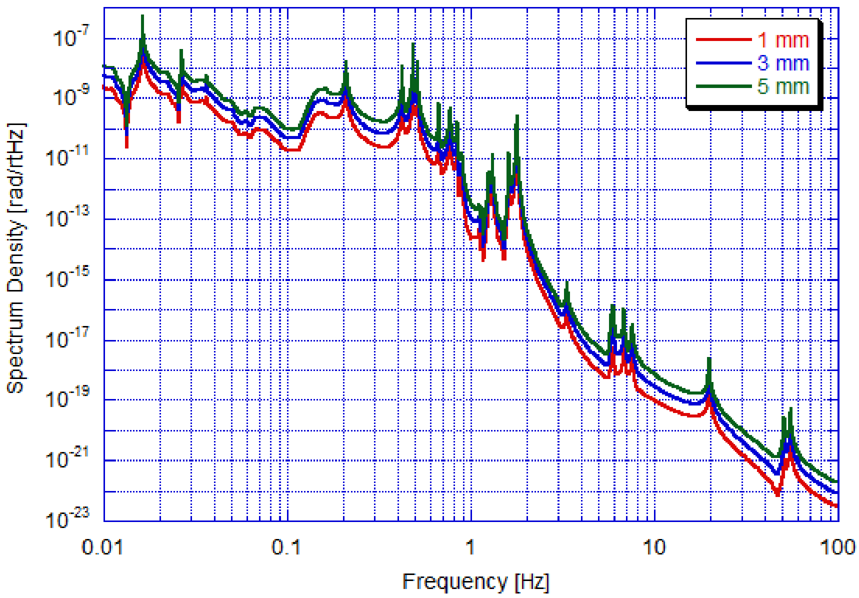
(2) Wire length

Graph 14 and 15 show angular mode amplitude of the TM, when one of the wires suspending the IM is a little longer than other wires. Compared to the geometric asymmetry case, pitch and yaw motions excited by this asymmetry are relatively small.

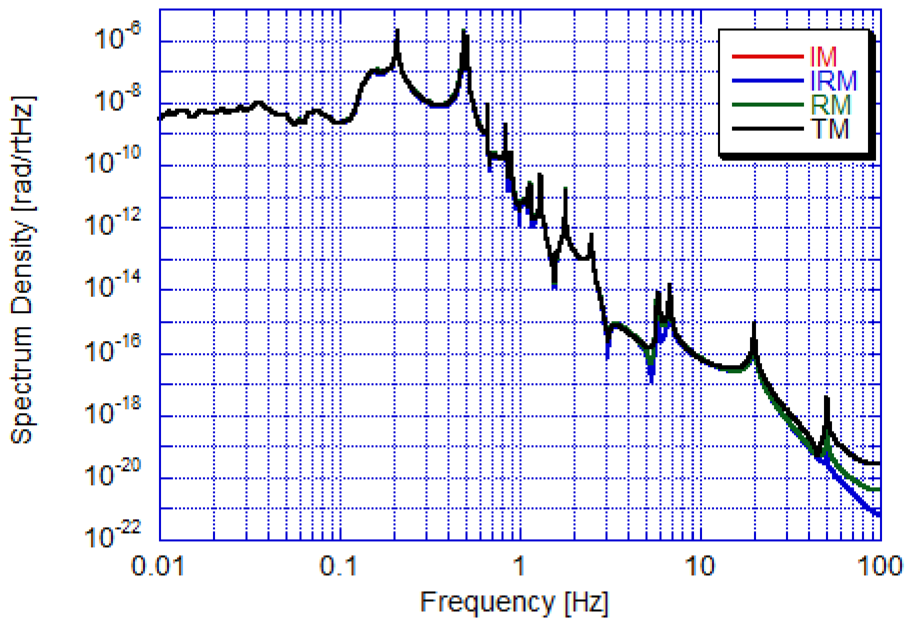
Graph 16 and 17 show the cases that there is wire length asymmetry of 1 mm in IM, IRM, RM and TM suspension. In high frequencies, the effect of asymmetry is more critical when the asymmetry places nearer to the TM.



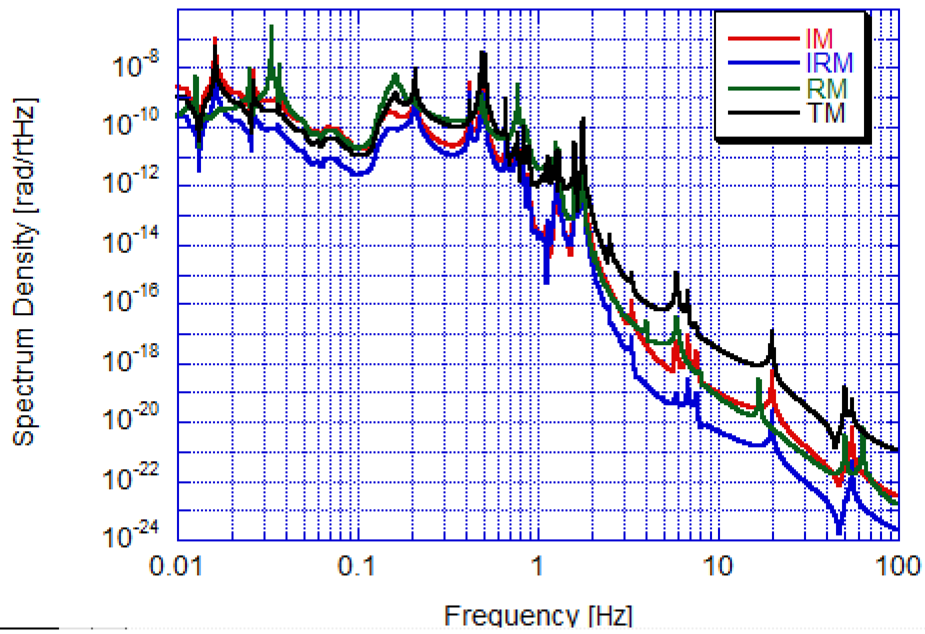
Graph14: Pitch motion of the TM with asymmetry in wire length of IM suspension



Graph15: Yaw motion of the TM with asymmetry in wire length of IM suspension



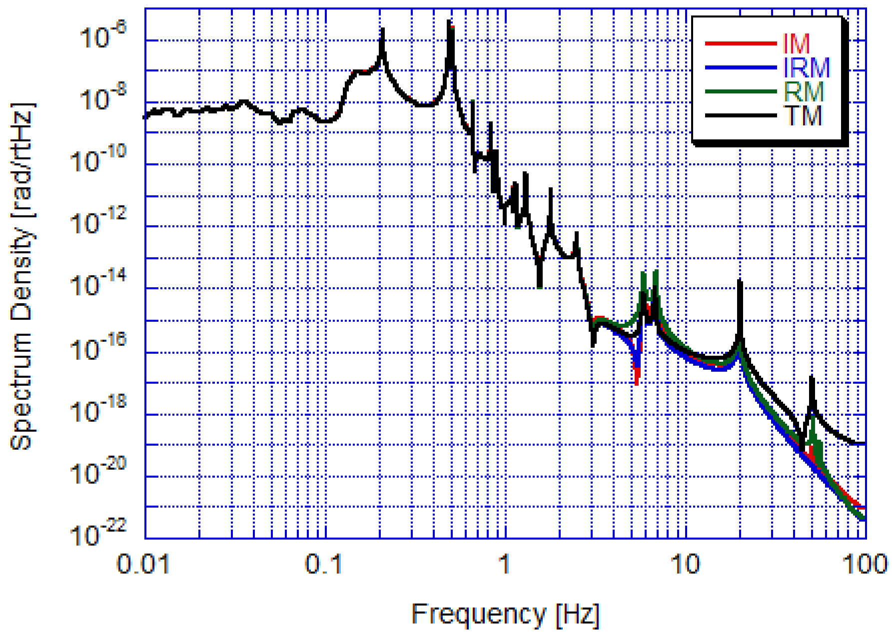
Graph16: Pitch motion of the TM with asymmetry (1 mm) in IM, IRM, RM and TM suspension



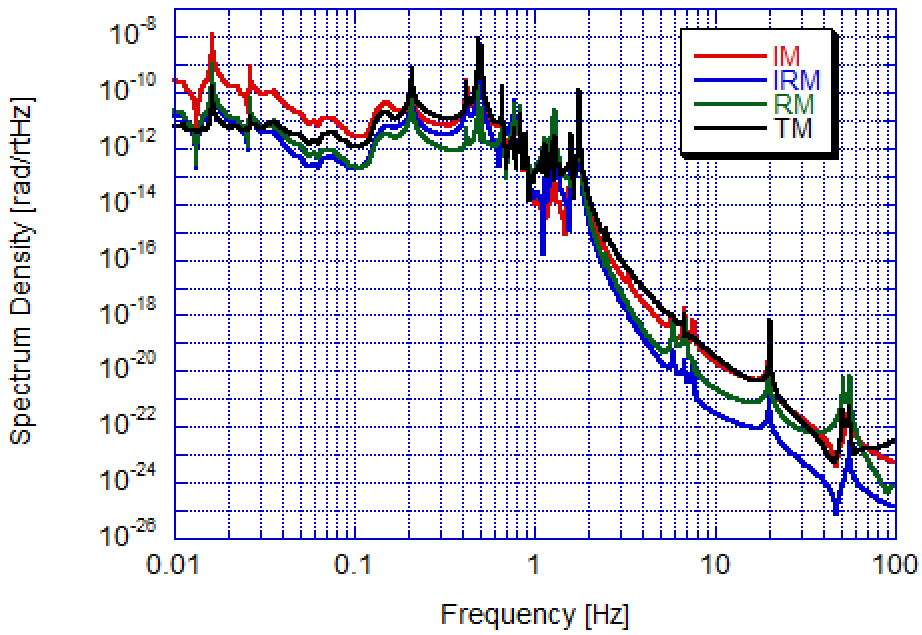
Graph17: Yaw motion of the TM with asymmetry (1 mm) in IM, IRM, RM and TM suspension

(3) Wire Diameter

Graph 18 and 19 show the angular motion of the TM, in the case that one of the wires is 5% thicker than other wires.



Graph18: Pitch motion of the TM with asymmetry in wire diameter of IM, IRM, RM and TM suspension

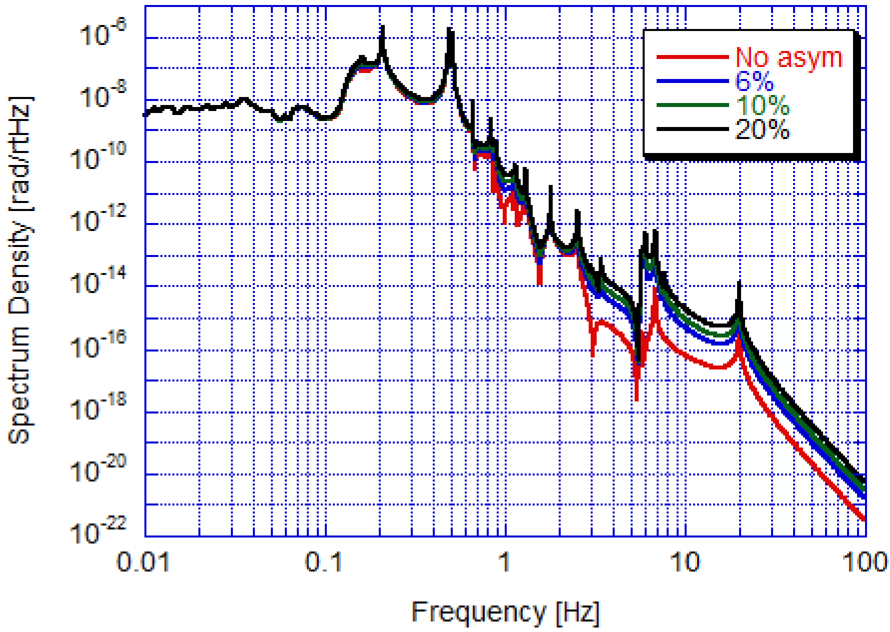


Graph19: Yaw motion of the TM with asymmetry in wire diameter of IM, IRM, RM and TM suspension

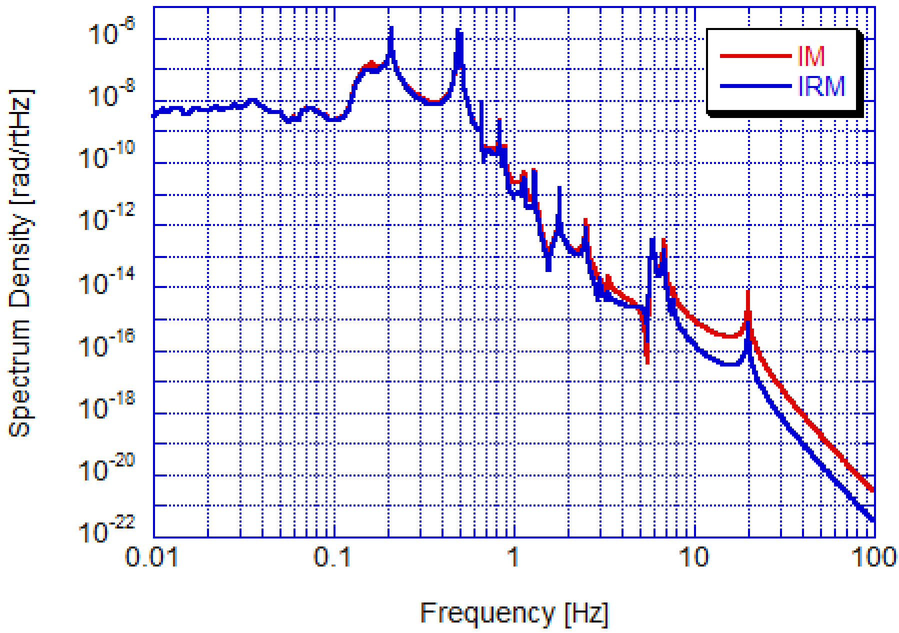
(4) Stiffness of mini-GAS filters

In the IM and IRM suspension, all the wires are attached to mini-GAS filters, in order to attenuate the vertical seismic noise that is introduced to the PF via heat links. As each mini-GAS filter has individual characteristic, it is difficult to tune all the filters at the same resonant frequencies. In the following simulation, it is assumed that one of the mini-GAS filters is stiffer than other filters. If there is asymmetry in stiffness, vertical motions are coupled with pitch and roll motions.

Graph 20 shows the pitch motion of the TM with asymmetry in stiffness of mini-GAS for the IM suspension. Graph 21 shows the comparison of the two cases that asymmetry is the IM and IRM suspension.



Graph20: Pitch motion of the TM with asymmetry in the stiffness of mini-GAS filters (of the IM suspension)



Graph21: Pitch motion of the TM with asymmetry in the stiffness of mini-GAS filters
(of the IM and MB suspension, 10% asymmetry in the stiffness)

(5) Summary

* These kinds of asymmetry may allow large seismic excitation in higher frequencies. Especially, the asymmetry in the TM suspension will be the most critical.
* Asymmetry in 4-wire suspensions should be treated more delicately, because if we assume too large asymmetry, the wires may be loose and do diode-like behavior. But we can notice it easily, so that we will not allow so large asymmetry in the 4-wire suspension systems. The asymmetry assumed in the above simulation may be too large.

*** Attachment Point of the Heat Link**

In the above simulation, it is assumed that the heat link is attached to the PF at its CoM. In the following simulation, we assume that the attachment position and the CoM of the PF are misaligned. If they are misaligned in the vertical direction, horizontal and pitch/roll motions are coupled (Graph 22), and if they are misaligned in the horizontal direction, vertical and pitch motions, horizontal and yaw motions are coupled (Graph 23 and 24).

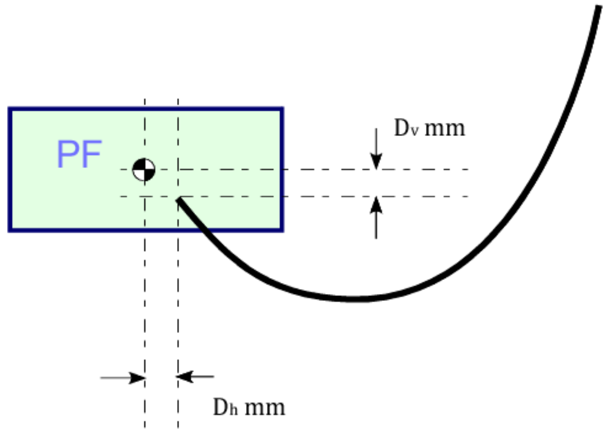
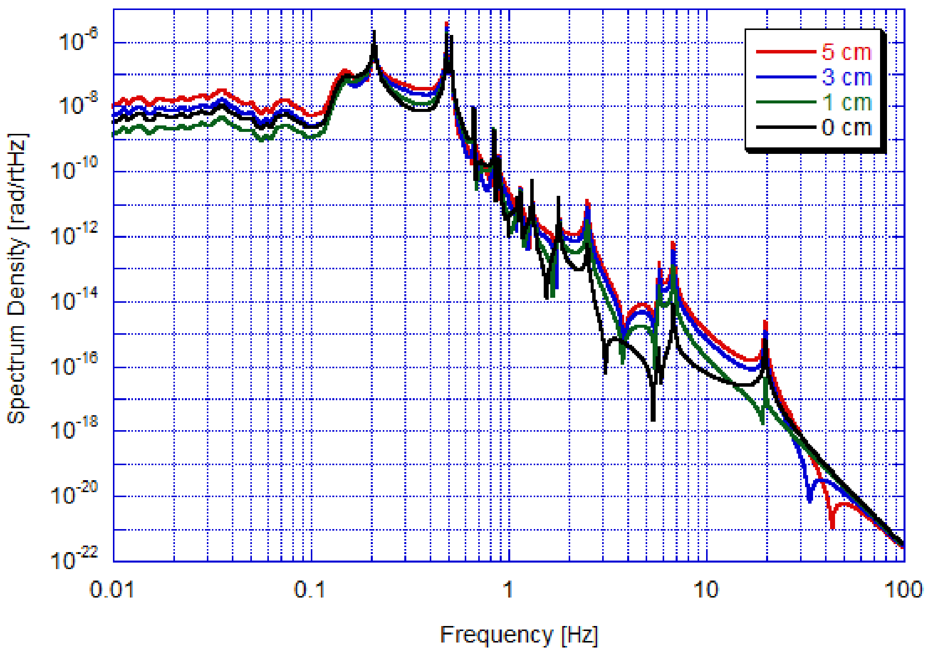
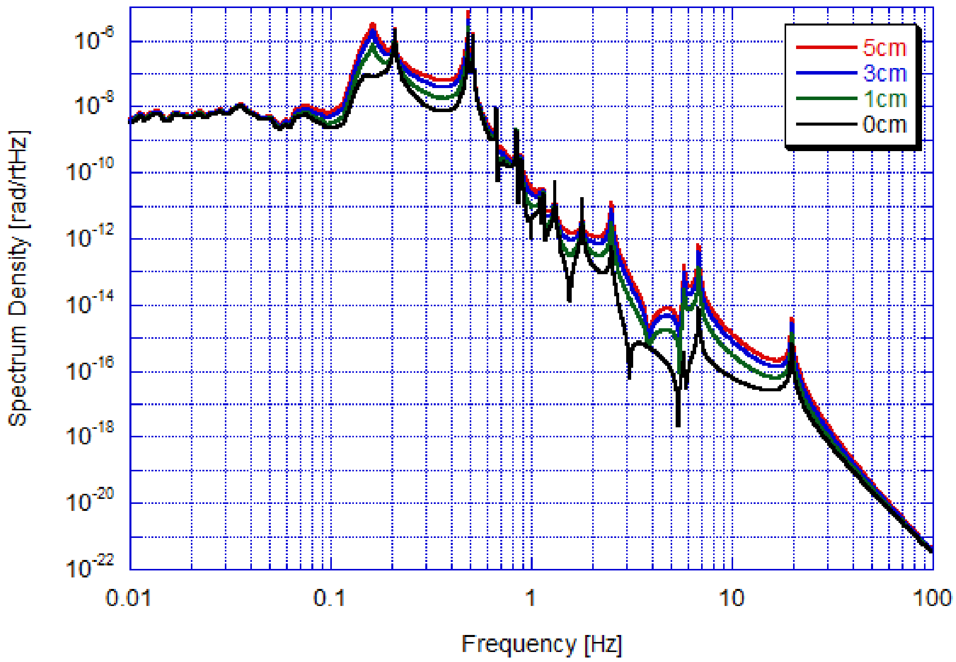


Figure 9: Attachment point of the heat link

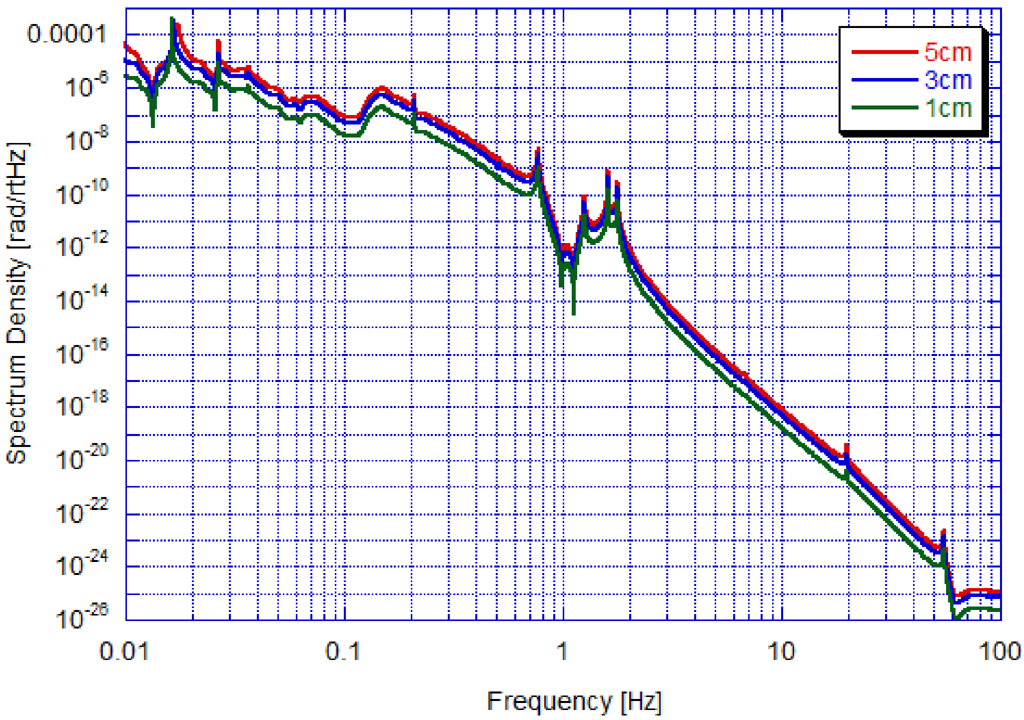
The misalignment of the attachment point affects the angular motions of the TM quite largely. Especially, it seems that the misalignment in the horizontal direction produces large angular fluctuation. If we allow 5 cm misalignment in the horizontal direction, the rms amplitude of the yaw motion reaches more than 10^{-5} rad. We should be very careful when we attach the heat link to the PF, so that it grabs the heat link in the CoM.



Graph22: Pitch motion of the TM when the heat link is clamped below the CoM (D_v=1, 3, 5 cm)



Graph23: Pitch motion of the TM when the heat link is clamped aside the CoM (D_h=1, 3, 5 cm)



Graph24: Yaw motion of the TM when the heat link is clamped aside the CoM (D_h=1, 3, 5 cm)

*** Angular Fluctuation: Summary**

Graph25-28 show the angular motion of the TM with the following asymmetry.

1. IP: Asymmetry in the stiffness of the IPs (±0.2%, IPs are tuned at 30 mHz)
2. Misalign: Misalignment of the suspension points at F4 (10 mm in z-direction)
3. Geometric: Geometric asymmetry in the TM suspension (5 mm in z-direction)
4. Wire Diam: Asymmetry in wire diameters in the TM suspension (one wire is 5% thicker)
5. Wire Length: Asymmetry in wire length in the TM suspension (one wire is 3 mm longer)
6. mini-GAS: Asymmetry in the stiffness of mini-GAS filters (20% in stiffness)
7. Heat Link: Misalignment of the attachment point of the heat link (1cm in the z-direction)
8. No Asym: No asymmetry

Pitch Mode:

* In high frequencies, the asymmetry in the geometry, and wire length & diameter of the TM suspension are the main contributor. The asymmetry in higher stages is not a problem, because it is cut off by the mechanical filters.

* In low frequencies, the misalignment of the heat link attachment is the main contributor.

* In any case, the rms amplitude of the pitch is estimated as several 10^{-7} rad. The main contributor to the rms is the resonance peaks at 0.2 and 0.5 Hz. They are both corresponding to the pendulum modes, which can be damped by passive eddy current damping (discussed later) and active controls.

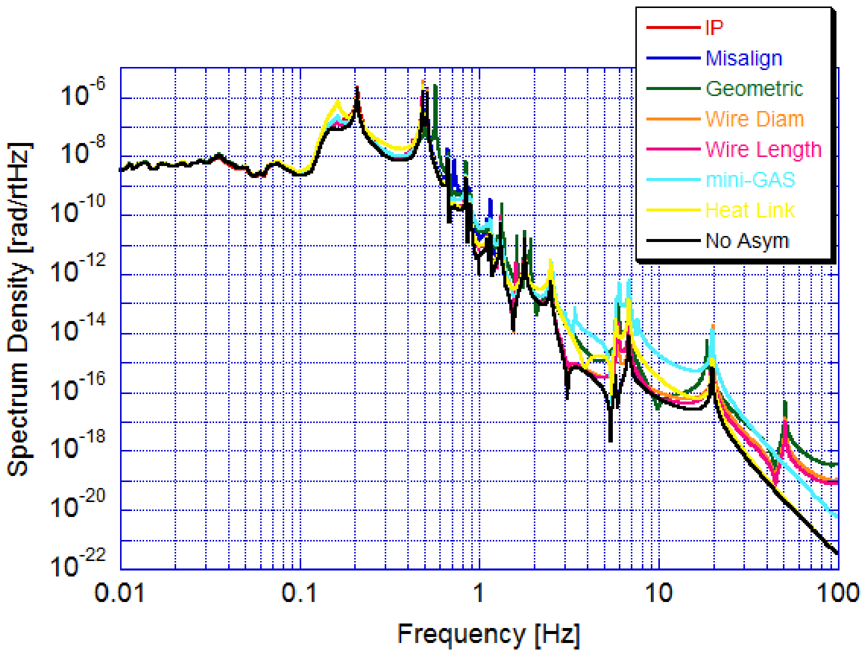
Yaw Mode:

* In high frequencies, the asymmetry in the geometry, and wire length & diameter of the TM suspension are the main contributor. The asymmetry in the last stage (TM suspension) is the most critical to the angular fluctuation in high frequencies.

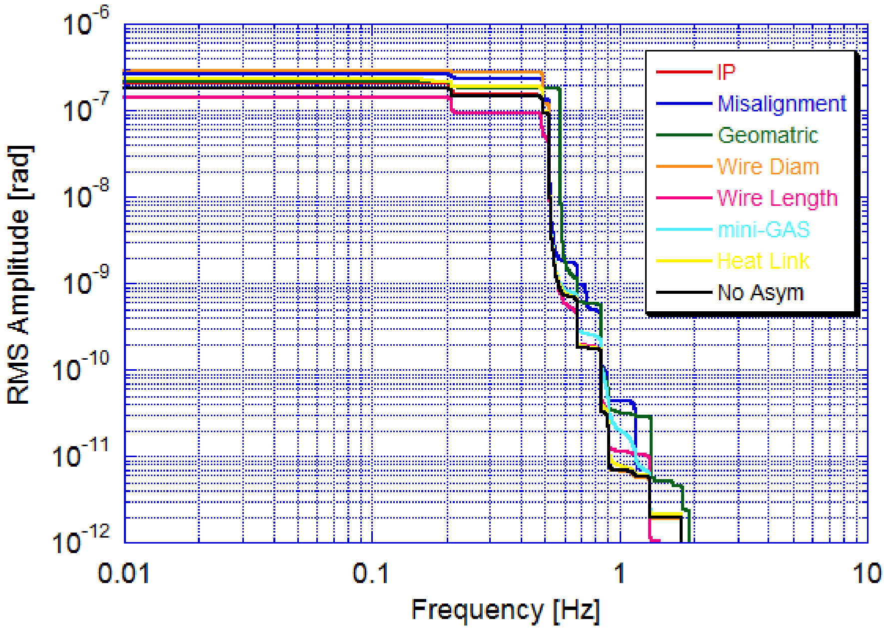
* In low frequencies, the misalignment of the heat link attachment, and the asymmetry in the stiffness of the IPs are the main contributor. If these kinds of asymmetry exist, the rms amplitude of the yaw reaches more than 10^{-6} rad.

* The main contributor to the rms is the torsion mode peaks at <50 mHz. Those peaks can be damped by the eddy current damping at the top of the stage.

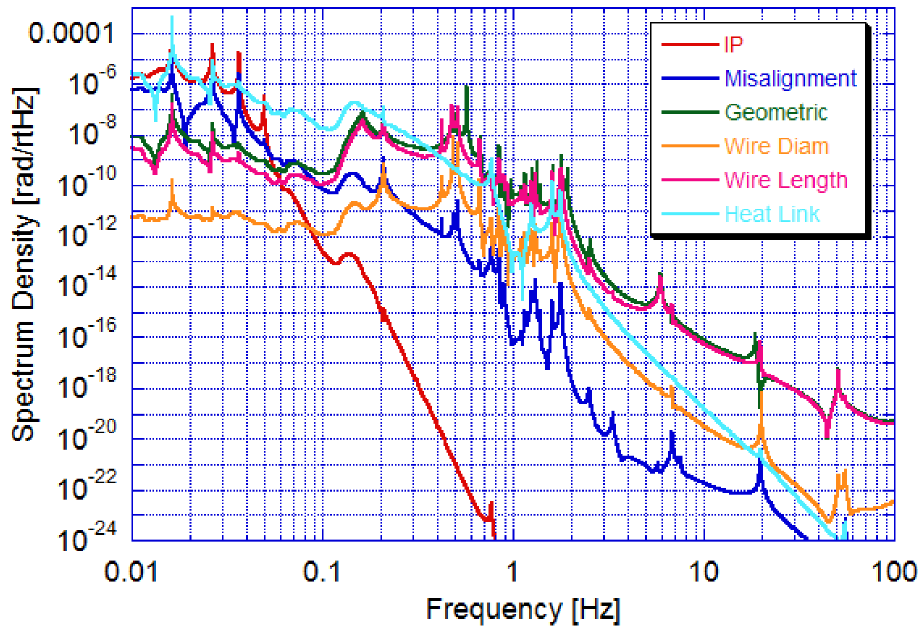
* Even though the torsion mode peaks may be effectively damped, the rms amplitude will still remain large. It is important not to allow large asymmetry in IPs, or large misalignment in the attachment point of the heat link.



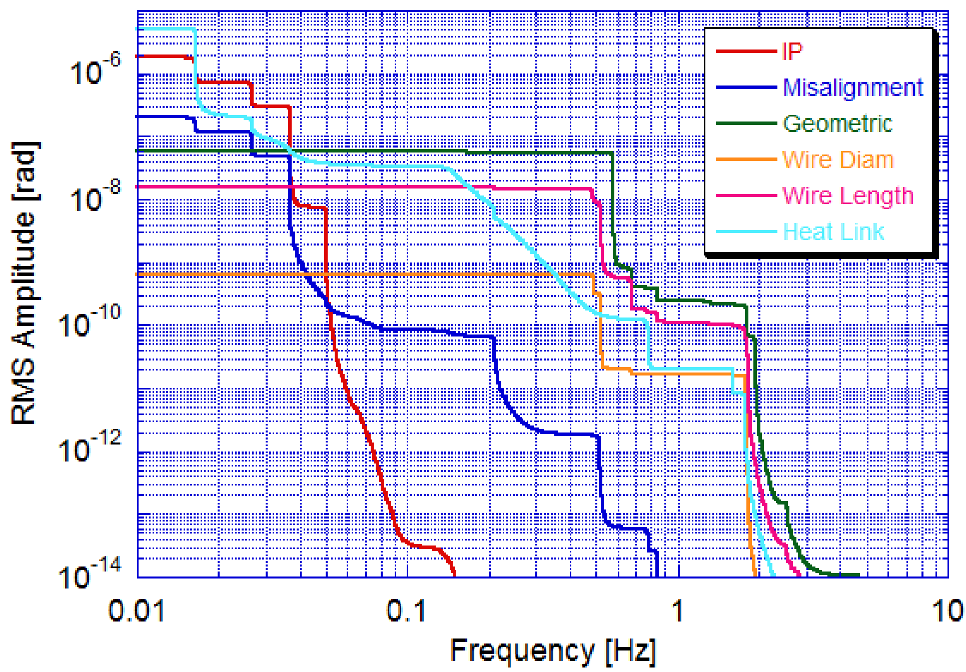
Graph 25: Pitch motion of TM with asymmetry in the system



Graph 26: Pitch motion of TM with asymmetry in the system (rms)



Graph 27: Yaw motion of TM with asymmetry in the system



Graph 28: Yaw motion of TM with asymmetry in the system (rms)

*** Effect of Eddy Current Damping**

Now Constructing..