Virtual Mach-Zehnder based on polarization separation for generating multiple sets of non-cascaded sidebands

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It's well known that light passing through a series of phase modulators, each driven at a unique frequency generates a spectrum that has not only the sidebands directly imposed by each modulator, but also sidebands of sidebands imposed by the modulators further downstream. It is also well known that an array of modulators in parallel, i.e. in opposite arms of a Mach-Zehnder only impose sidebands on the carrier without introducing sidebands of sidebands. At the LSC meeting in August 2004, Kentaro Somiya discussed the optical layout used to generate the desired input field for the 40m interferometer at CalTech. A Mach-Zehnder interferometer is used to allow sidebands at both modulation frequencies to be generated without sidebands of sidebands that would obscure control signals generated through double-demodulation. He also discussed a concern about differential phase noise of the Mach-Zehnder being problematic. What is described below is a "virtual Mach-Zehnder" based on polarization separation that is suitable for generating sidebands at two unique modulation frequencies but is immune to differential phase noise.

Consider the actual Mach-Zehnder configuration for generating the appropriate input light spectrum

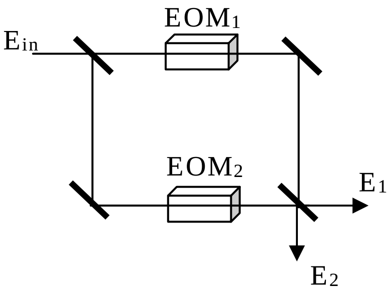

Figure 1 A real Mach-Zehnder interferometer with an EOMs in each arm

The output E_1 is

$$E_1 = \frac{i}{2}\left(e^{ikn(t)L_1+\phi_1} + e^{ikn'(t)L_2+\phi_2}\right)E_{in} = e^{i(\phi_1+\phi_2)/2}e^{i(n(t)L_1+n(t)L_2)/2}\cos\left(\frac{\left(n(t)L_1 - n(t)L_2\right)+\left(\phi_1-\phi_2\right)}{2}\right)E_{in}$$

Where n(t) and L_1 are the index of refraction and length of EOM1 respectively, n'(t) and L_2 are the index of refraction and length of EOM2 respectively, and ϕ_1 and ϕ_2 are the static phases accumulated during propagation in each arm. n(t) and n'(t) are responsible for the RF modulation, and the control scheme fixes ϕ_1 and ϕ_2 by feeding back to one of the mirror positions to set $\phi_1-\phi_2=0$ so that the field E_1 is bright and the modulated is purely phase modulation.

Consider the optical layout shown below

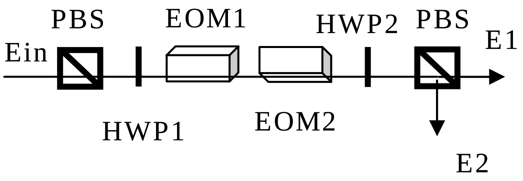

Figure 2. A virtual Mach-Zehnder with an EOM for each polarization.

The Jones matrix for the optical path is

$$M = -\frac{1}{2}\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}e^{ikn'_e(t)L_1} & 0\\ 0 & e^{ikn'_oL_1}\end{bmatrix}\begin{bmatrix}e^{ikn_oL_2} & 0\\ 0 & e^{ikn_e(t)L_2}\end{bmatrix}\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$$

Where the phase modulation is only on the extraordinary ray, not the ordinary ray in the phase modulators. The output E_1 is given by

$$\vec{E}_1 = M\begin{bmatrix}E_{in,p}\\ E_{in,s}\end{bmatrix} = -\frac{1}{2}\left(e^{ikn'_e(t)L_1+ikn_oL_2} + e^{ikn_e(t)L_2+ikn'_oL_1}\right)E_{in,s}$$

Which is identical to that of the real Mach-Zehnder interferometer when the static phases are related by

$$kn_oL_2 \rightarrow \phi_1$$
$$kn'_oL_1 \rightarrow \phi_2$$

This phase difference cannot be adjusted by moving a mirror, but rather, if necessary can be adjusted by inserting a waveplate with the proper birefringence into the path. If, however, the length and crystal material of the two modulators is identical, this shouldn't be necessary. Note that because the differential phase shift cannot be adjusted by moving a mirror, differential phase noise cannot be introduced by moving a mirror either. The analogy between the real and the virtual Mach-Zehnder interferometers is easy to understand physically: In the real Mach-Zehnder of Figure 1 a beam splitter separates a beam into paths each path contains a beam of equal amplitude. An EOM in each arm modulates the light in that arm and the two paths are recombined by a second beamsplitter. The static length difference between the arms must be set to an integer number of wavelengths to keep the output on the bright fringe. In the virtual Mach-Zehnder of Figure 2, a half-waveplate separates a linearly polarized beam into two polarization components of equal amplitude. A vertical EOM and a horizontal EOM each modulate one of the polarization components and the two components are recombined by another half-waveplate and a polarizing beamsplitter. The static birefringence between the arms must be set to an integer number of wavelengths to keep the output on the bright fringe.

Clearly the virtual Mach-Zehnder and the real Mach-Zehnder are analogous. Differential phase noise in the real Mach-Zehnder is relatively easy to excite because it only requires the differential motion of the mirrors to modulate the path length difference. In the virtual Mach-Zehnder, however, differential phase noise is hard to excite because it requires differential changes in the birefringence, but there is no mechanism to generate this.