ROC error assessment

LCGT を構成するミラーおよびレンズには必ず研磨誤差が生ずる。
ここでは各光学素子における曲率半径 (Radius Of Carvature) の誤差により
gaussian beam の mode matching がどの程度悪化するかについて評価を行った。

1. **Arm cavity ($g_1=g_2=+0.5783$)**
2. **Arm cavity ($g_1=g_2=-0.5783$)**
3. **No-lens PRC design**
4. **Folded PRC with $g_1=g_2=+0.5783$**
5. **Folded PRC with $g_1=g_2=-0.5783$**

Section 1: Arm cavity
with $g_1=g_2=+1/\mathrm{sqrt}(3) = 0.57735$

ITM **ETM**

Arm cavity

L_{34} = 3006.69 m
R_1 = 7113.9
R_2 = 7113.9

d = 1503.345 (waist position)
ZR = 2904.238350 (Rayleigh range)

(mode mis-matching) =

$$1 - 4\left|\frac{z_{\mathrm{RC}}\sqrt{z_{\mathrm{Rx}}z_{\mathrm{Ry}}}}{[z_{\mathrm{RC}}+z_{\mathrm{Rx}}+i(d_{\mathrm{x}}-d_{\mathrm{C}})][z_{\mathrm{RC}}+z_{\mathrm{Ry}}+i(d_{\mathrm{y}}-d_{\mathrm{C}})]}\right|$$

Eigen mode of the cavity

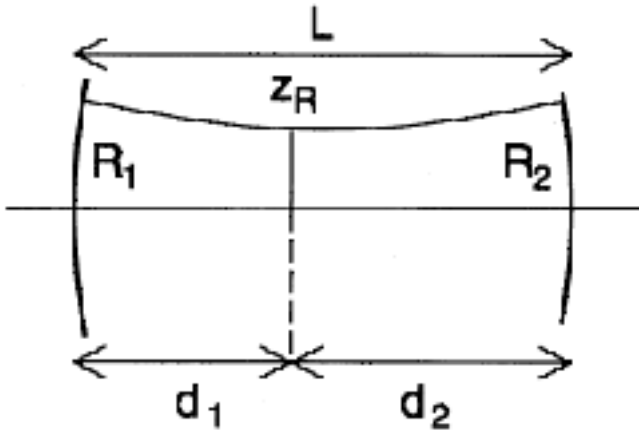
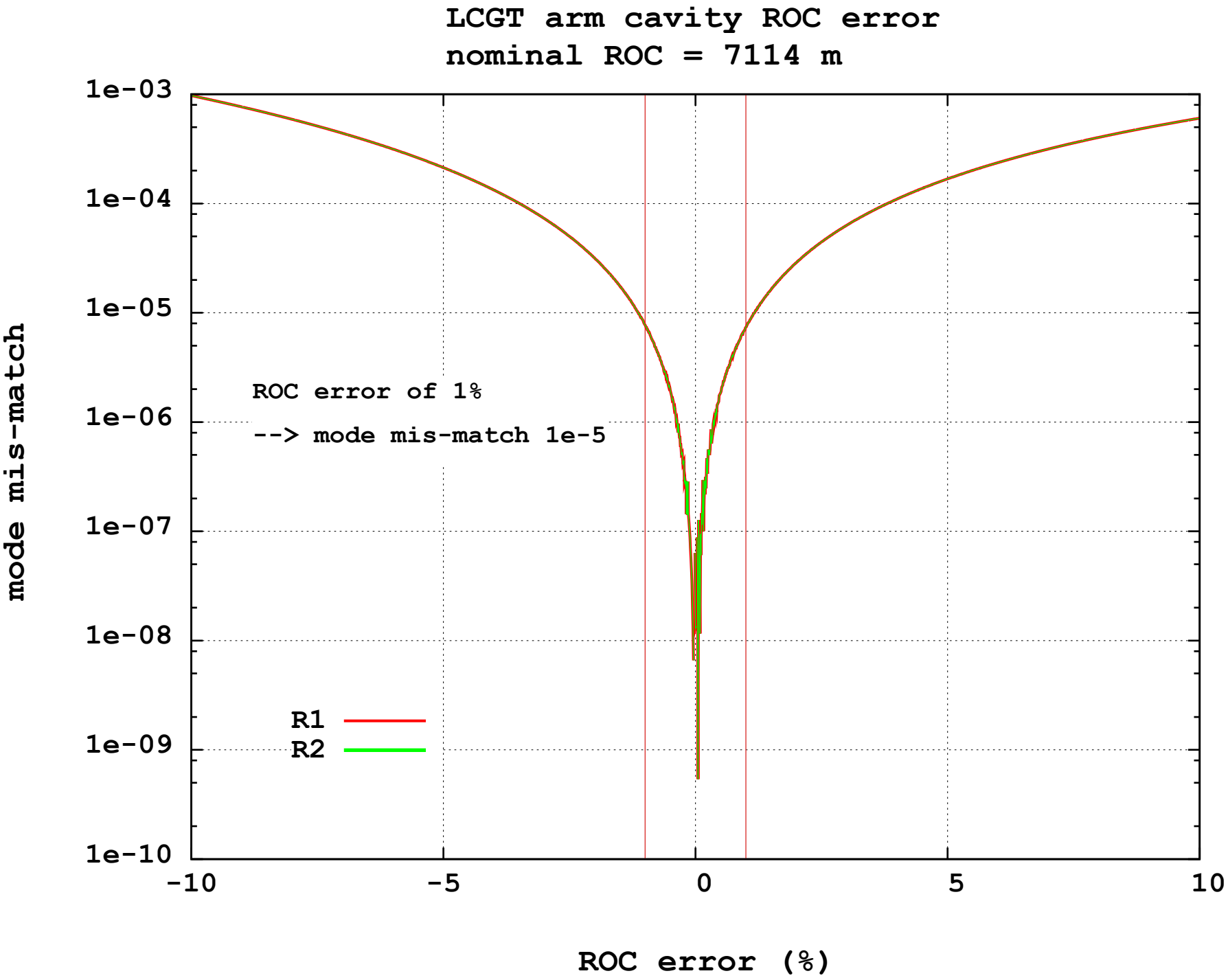


図 A-5 球面鏡によって作られる共振器の固有モード.

$$z_R{}^2 = \frac{L(R_1-L)(R_2-L)(R_1+R_2-L)}{(R_1+R_2-2L)^2}$$
$$= \frac{L^2 g_1 g_2(1-g_1g_2)}{(g_1+g_2-2g_1g_2)^2}$$
$$d_i = L\frac{(R_1-L)(R_2-L)}{(R_i-L)(R_1+R_2-2L)}$$
$$= L\frac{(1-g_i)}{g_i}\frac{g_1g_2}{g_1+g_2-2g_1g_2} \quad (i=1,2)$$


LCGT arm cavity ROC error
nominal ROC = 7114 m
mode mis-match
1e-03
1e-04
1e-05
1e-06
1e-07
1e-08
1e-09
1e-10
-10
-5
0
5
10
ROC error (%)
ROC error of 1%
--> mode mis-match 1e-5
R1
R2

ROC common and differential error

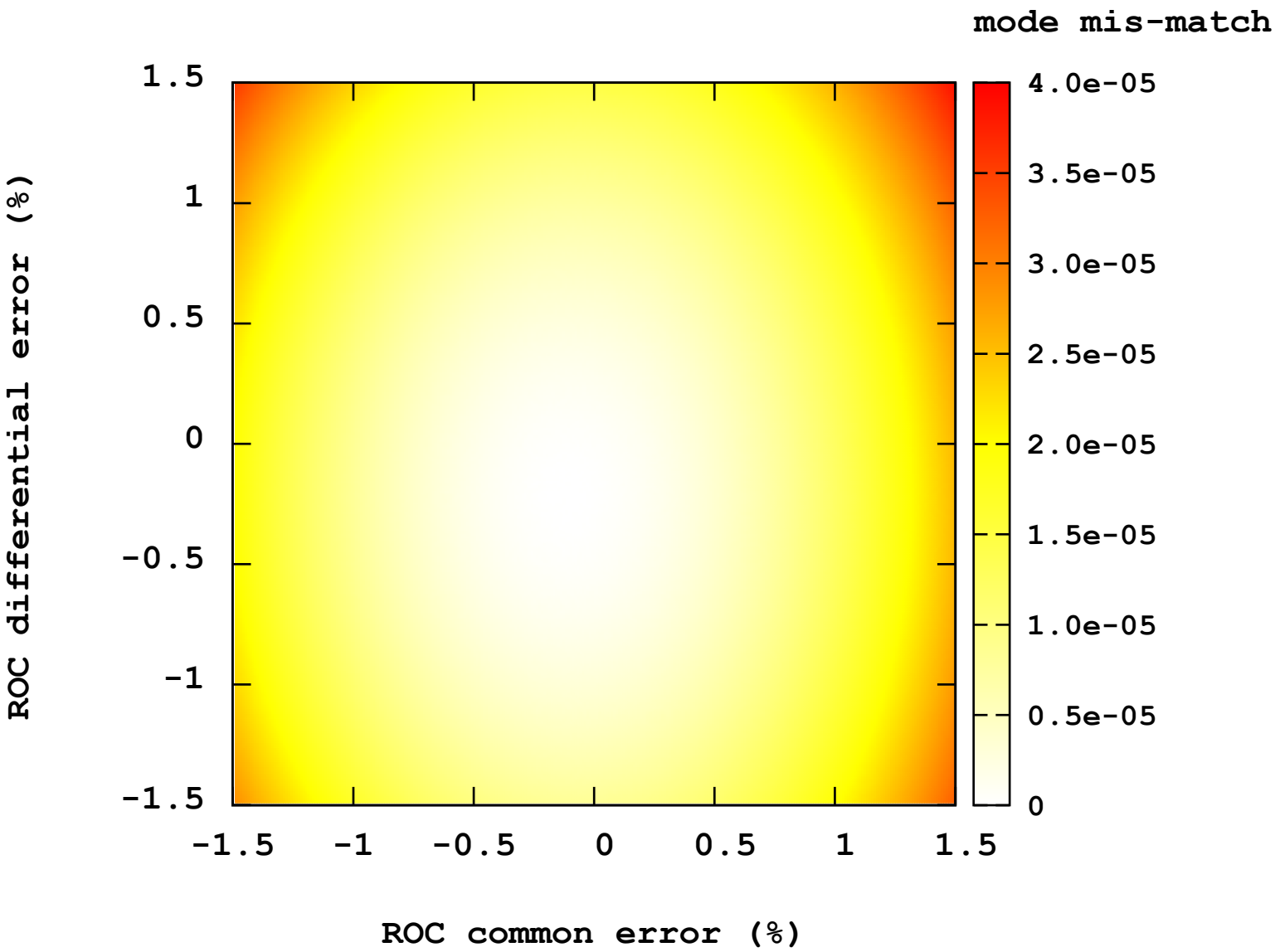
Since both of front and end mirror has same nominal ROCs, common ROC error only make a change of beam size at the waist. The waist position of the cavity mode does not change.
Here I evaluate the mode mis-match as a function of common and differential ROC error of the mirrors.

Contrary to my expectation, there is no significant difference between common and differential ROC errors.

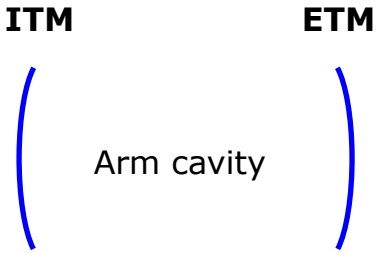
Arm cavity
with $g_1=g_2=+1/\mathrm{sqrt}(3) = 0.57735$

L_{34} = 3006.69 m
R_1 = 7113.9
R_2 = 7113.9

Z = 1503.345 (waist position)
ZR = 2904.238350 (Rayleigh range)

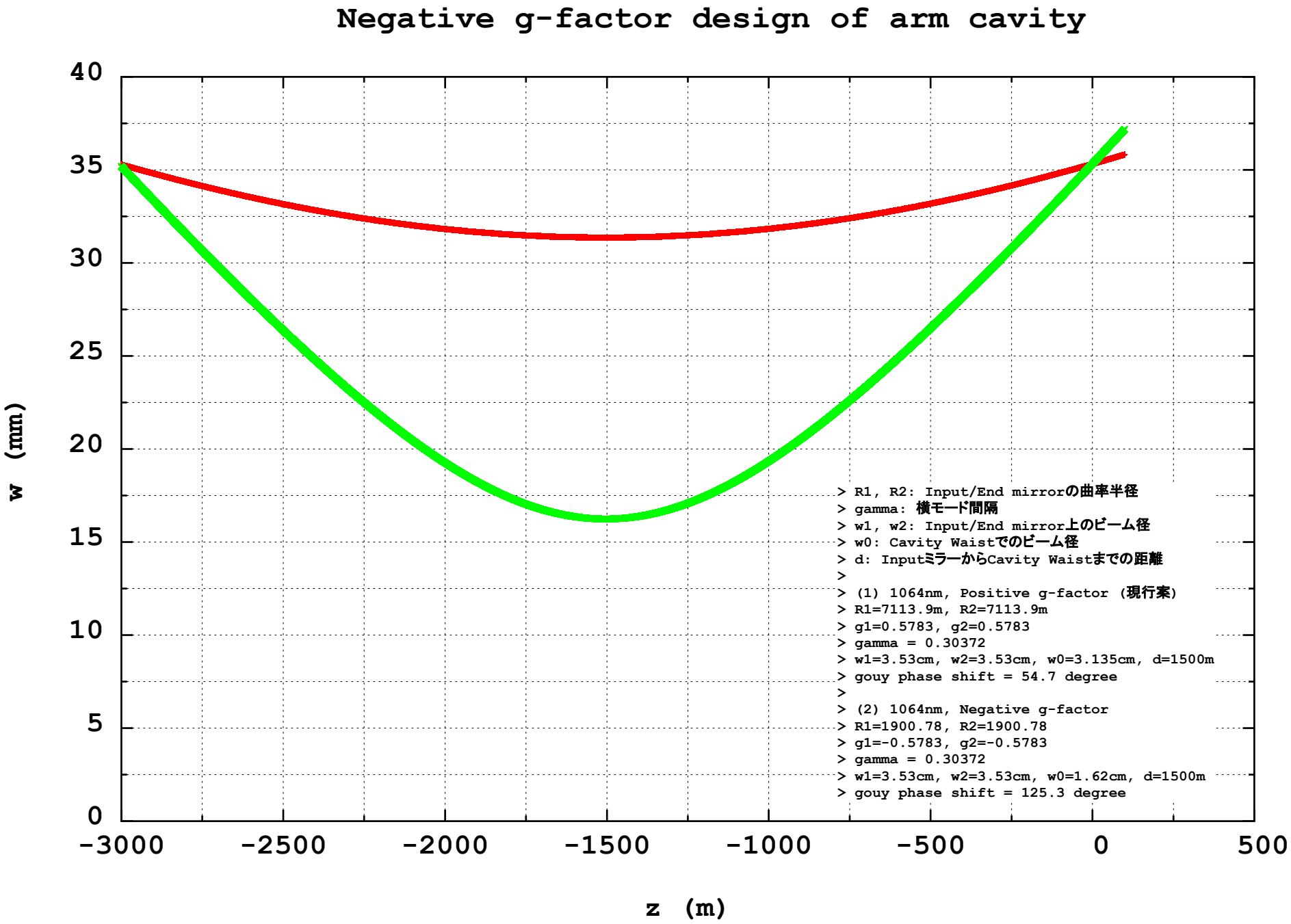


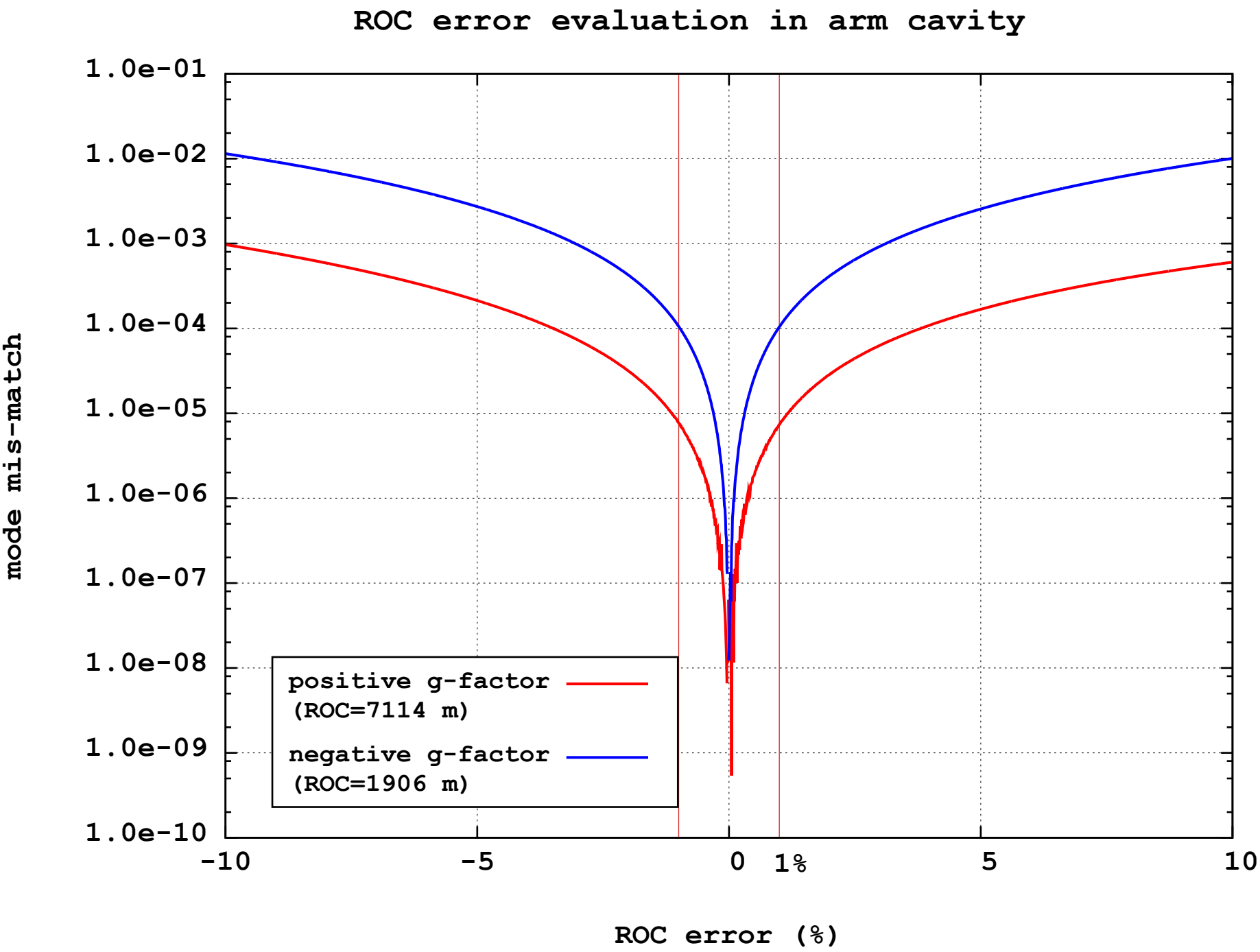
Section 2: Arm cavity
with $g_1=g_2=-1/\mathrm{sqrt}(3) = -0.57735$



L_{34} = 3006.69 m
R_1 = 1906.225
R_2 = 1906.225

d = 1503.345 (waist position)
ZR = 778.247313 (Rayleigh range)

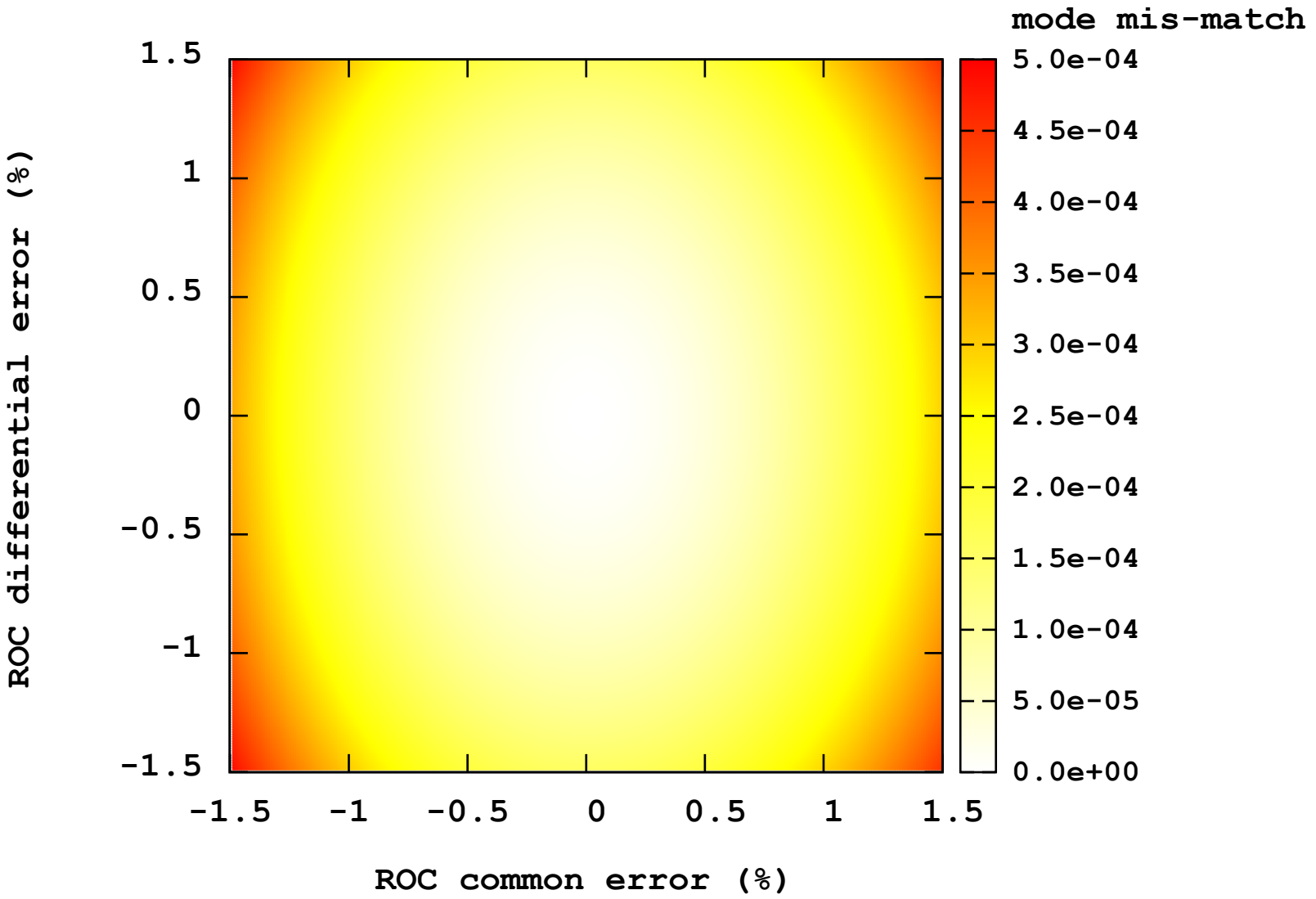

Negative g-factor design of arm cavity
w (mm)
z (m)
40
35
30
25
20
15
10
5
0
-3000
-2500
-2000
-1500
-1000
-500
0
500
> R1, R2: Input/End mirrorの曲率半径
> gamma: 横モード間隔
> w1, w2: Input/End mirror上のビーム径
> w0: Cavity Waistでのビーム径
> d: InputミラーからCavity Waistまでの距離
>
> (1) 1064nm, Positive g-factor (現行案)
> R1=7113.9m, R2=7113.9m
> g1=0.5783, g2=0.5783
> gamma = 0.30372
> w1=3.53cm, w2=3.53cm, w0=3.135cm, d=1500m
> gouy phase shift = 54.7 degree
>
> (2) 1064nm, Negative g-factor
> R1=1900.78, R2=1900.78
> g1=-0.5783, g2=-0.5783
> gamma = 0.30372
> w1=3.53cm, w2=3.53cm, w0=1.62cm, d=1500m
> gouy phase shift = 125.3 degree


ROC error evaluation in arm cavity
mode mis-match
1.0e-01
1.0e-02
1.0e-03
1.0e-04
1.0e-05
1.0e-06
1.0e-07
1.0e-08
1.0e-09
1.0e-10
-10
-5
0
1%
5
10
ROC error (%)
positive g-factor
(ROC=7114 m)
negative g-factor
(ROC=1906 m)

Arm cavity
with $g_1=g_2=-1/\sqrt{3} = -0.57735$

L_{34} = 3006.69 m
R_1 = 1906.2258
R_2 = 1906.2258

Z = 1503.345 (waist position)
ZR = 778.2373 (Rayleigh range)



Section 3: PRC cavity
with no-lens

$g_1 = g_2 = 1/\sqrt{3} = +0.57735$

PRC **ITM**

L_{34} = 73.3 m
R_1 = -7113.9
R_2 = 6926.359

d = 1503.345 (waist position)
ZR = 2904.238 (Rayleigh range)

$g_1 = g_2 = -1/\sqrt{3} = -0.57735$

PRC **ITM**

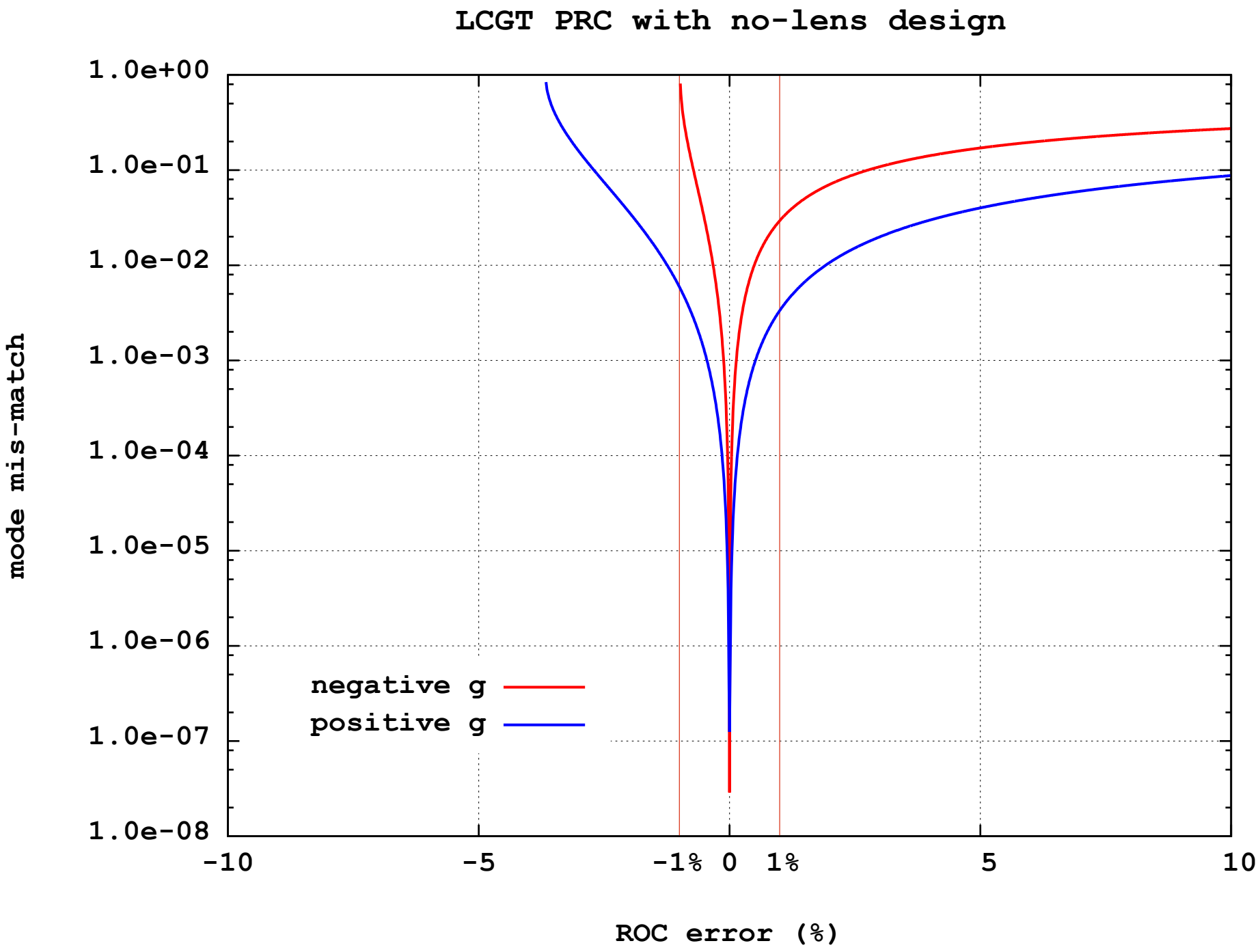
L_{34} = 73.3 m
R_1 = -1906.2258
R_2 = 1960.7954

d = 1503.345 (waist position)
ZR = 778.247 (Rayleigh range)

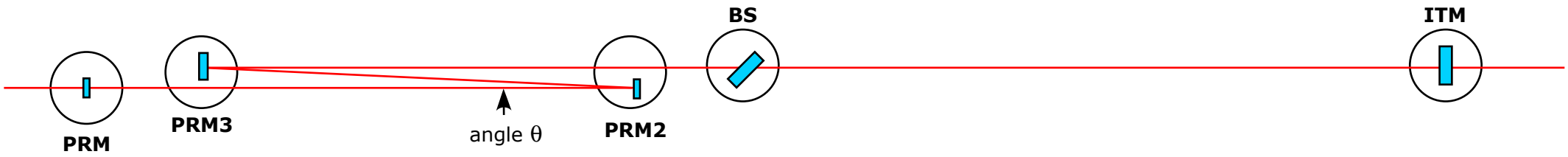
$R_1 + R_2 - L < 0$

$$\frac{-7113.90 + 6926.35 - 73.3}{-7113.90} = \frac{-260.85}{-7113.90} = 3.7\%$$

$$\frac{-1906.23 + 1960.80 - 73.3}{-1906.23} = \frac{-18.73}{-1906.23} = 0.98\%$$


LCGT PRC with no-lens design
mode mis-match
1.0e+00
1.0e-01
1.0e-02
1.0e-03
1.0e-04
1.0e-05
1.0e-06
1.0e-07
1.0e-08
-10
-5
-1%
0
1%
5
10
ROC error (%)
negative g
positive g

Section 4:
Folded PRC design with $g_1=g_2=+0.5783$



PRM | **ITM**

(PRC1 cavity) + beam expansion telescope (PRM2, PRM3) + (PRC2 cavity (

L_{12} = 17.1 + 2.34 m
R_1 = 337.0
R_2 = 269.2

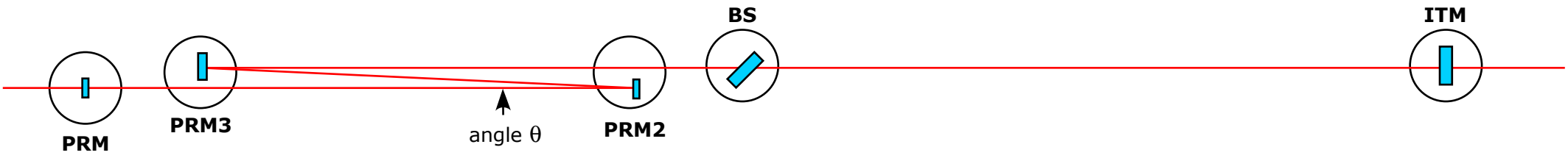
d = 8.5511655 (from PRM)
Z_R = 53.00

L_{23} = 14.1 m
f_2 = -1.95 m
f_3 = +16.0 m

L_{34} = 42.1 + 1695.16 m
R_3 = 5842.9
R_4 = -7113.9

d = 1503.345 (from ITM)
Z_R = 2904.23835

Folded PRC での cavity mode 計算



$$\mathrm{PRM}: \begin{pmatrix} 1 & 0 \\ -\frac{2}{337.0} & 1 \end{pmatrix}$$

$$\mathrm{PRM2}: \begin{pmatrix} 1 & 0 \\ -\frac{1}{-1.95} & 1 \end{pmatrix}$$

$$\mathrm{PRM3}: \begin{pmatrix} 1 & 0 \\ -\frac{1}{16.0} & 1 \end{pmatrix}$$

$$\mathrm{ITM}: \begin{pmatrix} 1 & 0 \\ -\frac{1}{-7113.9} & 1 \end{pmatrix}$$

$$L_{12}: \begin{pmatrix} 1 & 17.1 \\ 0 & 1 \end{pmatrix}$$

$$L_{23}: \begin{pmatrix} 1 & 14.1 \\ 0 & 1 \end{pmatrix}$$

$$L_{34}: \begin{pmatrix} 1 & 42.1 \\ 0 & 1 \end{pmatrix}$$

PRM2 から ITM までの往復部分を ABCD matrix で表現すると。

$$\mathrm{PRM2} \to \mathrm{L23} \to \mathrm{PRM3} \to \mathrm{L34} \to \mathrm{ITM} \to \mathrm{L34} \to \mathrm{PRM3} \to \mathrm{L23} \to \mathrm{PRM2}$$

$$= \begin{pmatrix} 0.982618 & 4.48482362 \\ -0.00742942 & 0.982618 \end{pmatrix}$$

これを距離 L だけ進んで、ROC が R のミラーで反射して戻ってきた場合の行列と解釈すると

$$= \begin{pmatrix} 1-\frac{2L}{R} & 2L-\frac{2L^2}{R} \\ -\frac{2}{R} & 1-\frac{2L}{R} \end{pmatrix}$$

$$R = 269.2000$$

$$L = 2.33966$$

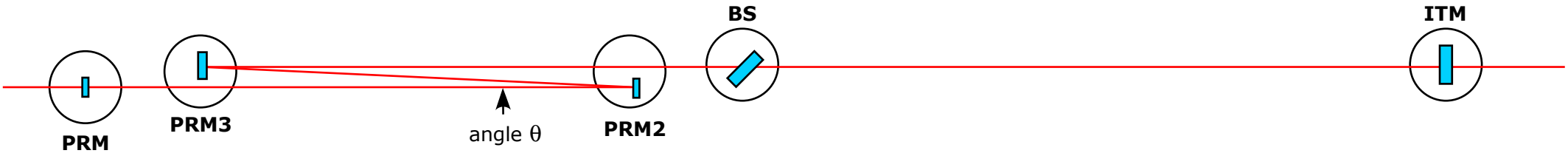
つまり下記のような cavity と思えばよい。

L_{12} = 17.1 + 2.34 m　(17.1m は PRM-PRM2 間の距離)

R_2 = 269.2

R_1 = 337.0　(PRM の ROC)

Folded PRC での cavity mode 計算



$$\text{PRM:}\begin{pmatrix} 1 & 0 \\ -\frac{2}{337.0} & 1 \end{pmatrix}$$

$$\text{PRM2:}\begin{pmatrix} 1 & 0 \\ -\frac{1}{-1.95} & 1 \end{pmatrix}$$

$$\text{PRM3:}\begin{pmatrix} 1 & 0 \\ -\frac{1}{16.0} & 1 \end{pmatrix}$$

$$\text{ITM:}\begin{pmatrix} 1 & 0 \\ -\frac{1}{-7113.9} & 1 \end{pmatrix}$$

$$L_{12}:\begin{pmatrix} 1 & 17.1 \\ 0 & 1 \end{pmatrix}$$

$$L_{23}:\begin{pmatrix} 1 & 14.1 \\ 0 & 1 \end{pmatrix}$$

$$L_{34}:\begin{pmatrix} 1 & 42.1 \\ 0 & 1 \end{pmatrix}$$

同様に、PRM3 から PRMM までの往復部分を ABCD matrix で表現すると。

$$\text{PRM3} \to \text{L23} \to \text{PRM2} \to \text{L12} \to \text{PRM} \to \text{L12} \to \text{PRM2} \to \text{L23} \to \text{PRM3}$$

$$= \begin{pmatrix} 0.4197540 & 2406.70721472 \\ -3.42296133 \times 10^{-4} & 0.4197540 \end{pmatrix}$$

これを距離 L だけ進んで、ROC が R のミラーで反射して戻ってきた場合の行列と解釈すると

$$= \begin{pmatrix} 1-\frac{2L}{R} & 2\,L-\frac{2d^2}{R} \\ -\frac{2}{R} & 1-\frac{2L}{R} \end{pmatrix}$$

$$R = 5842.8939$$

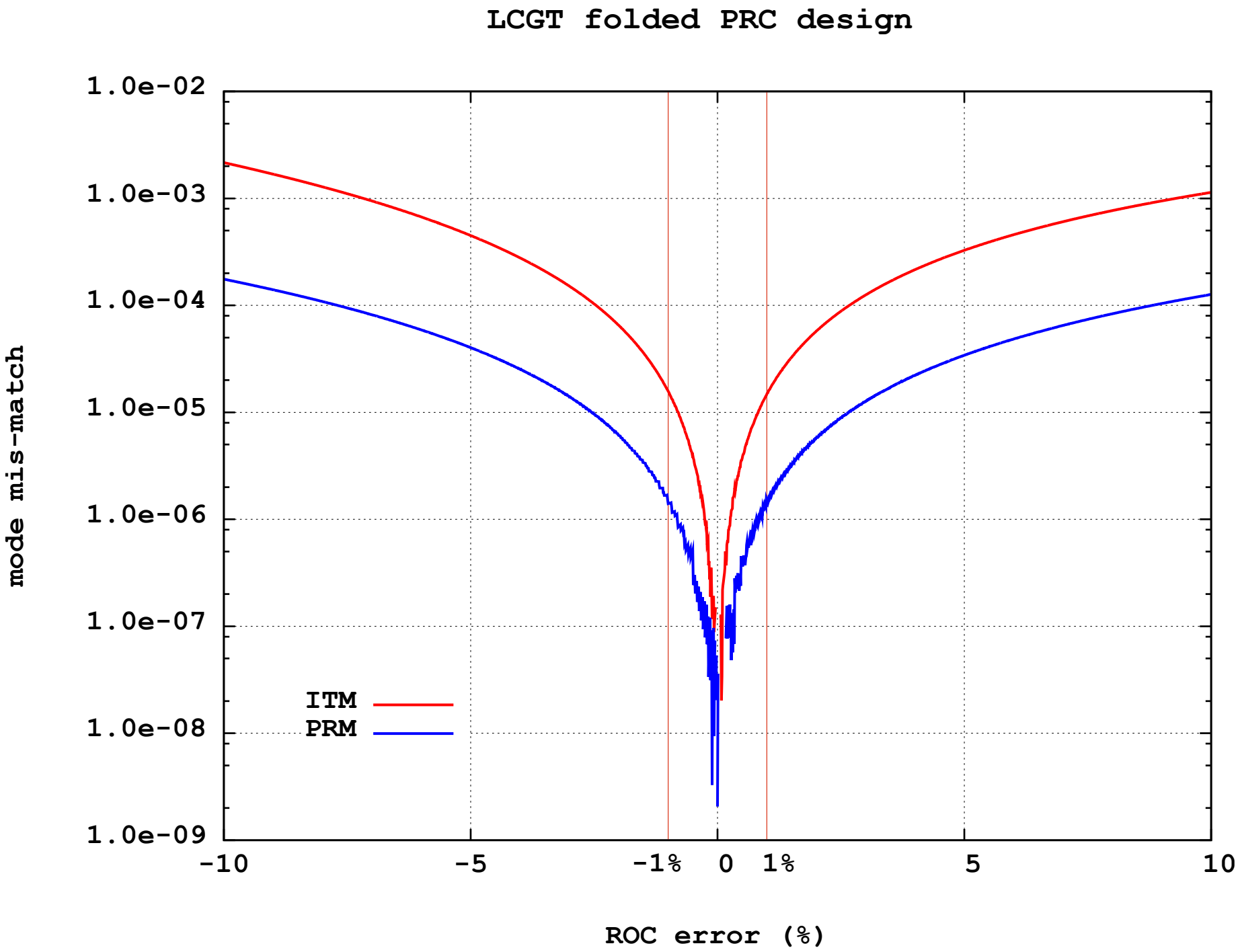
$$L = 1695.1579172$$

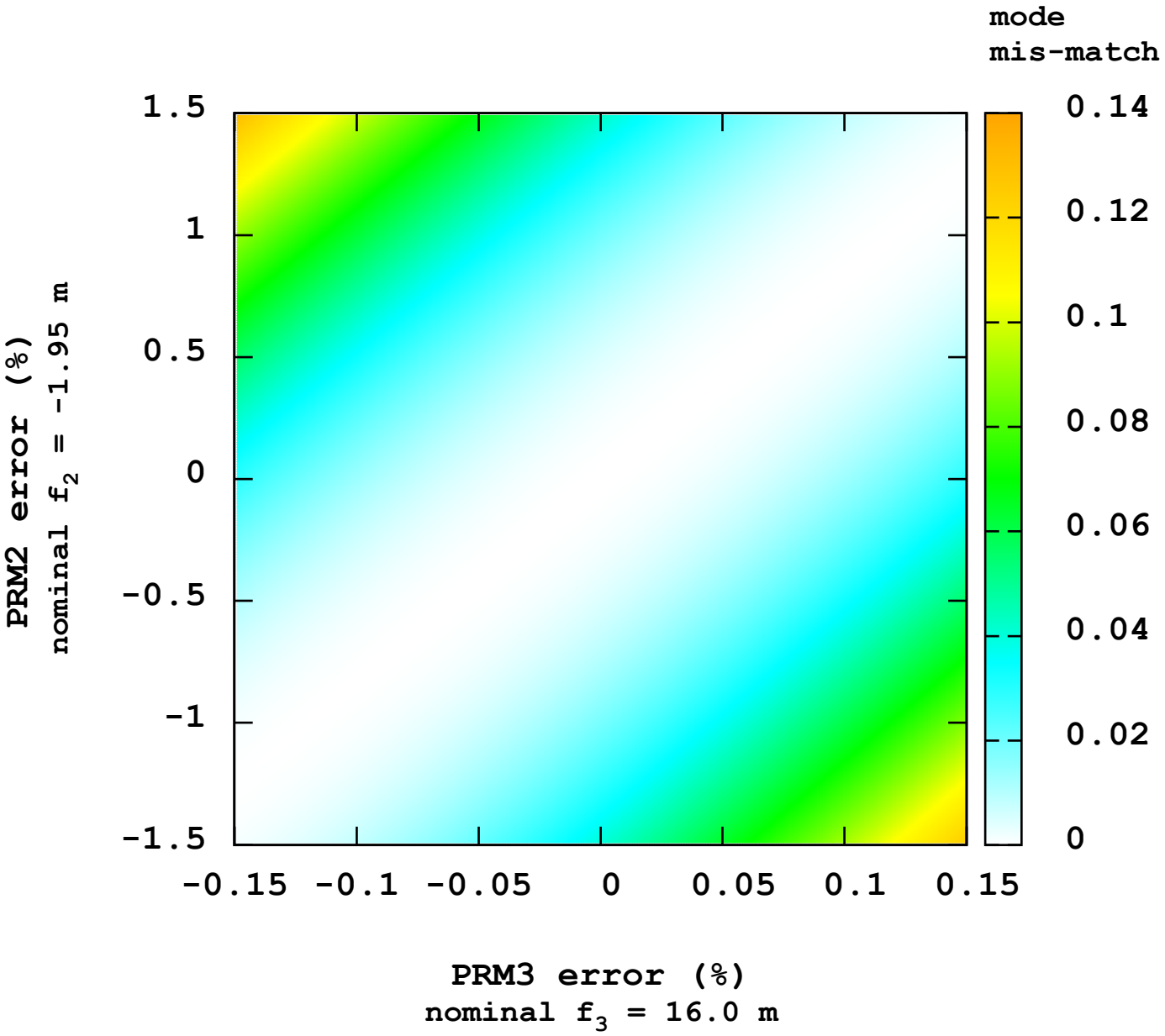
つまり下記のような cavity と思えばよい。

L_{34} = 42.1 + 1695.16 m　　(42.1m は PRM3-ITM 間の距離)

R_3 = 5842.9

R_4 = -7113.9　　(ITM の ROC)


LCGT folded PRC design
1.0e-02
1.0e-03
1.0e-04
1.0e-05
1.0e-06
1.0e-07
1.0e-08
1.0e-09
mode mis-match
ITM
PRM
-10
-5
-1%
0
1%
5
10
ROC error (%)


mode
mis-match
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1.5
1
0.5
0
-0.5
-1
-1.5
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
PRM2 error (%)
nominal f2 = -1.95 m
PRM3 error (%)
nominal f3 = 16.0 m

Beam expansion telescope の補正

レンズの合成

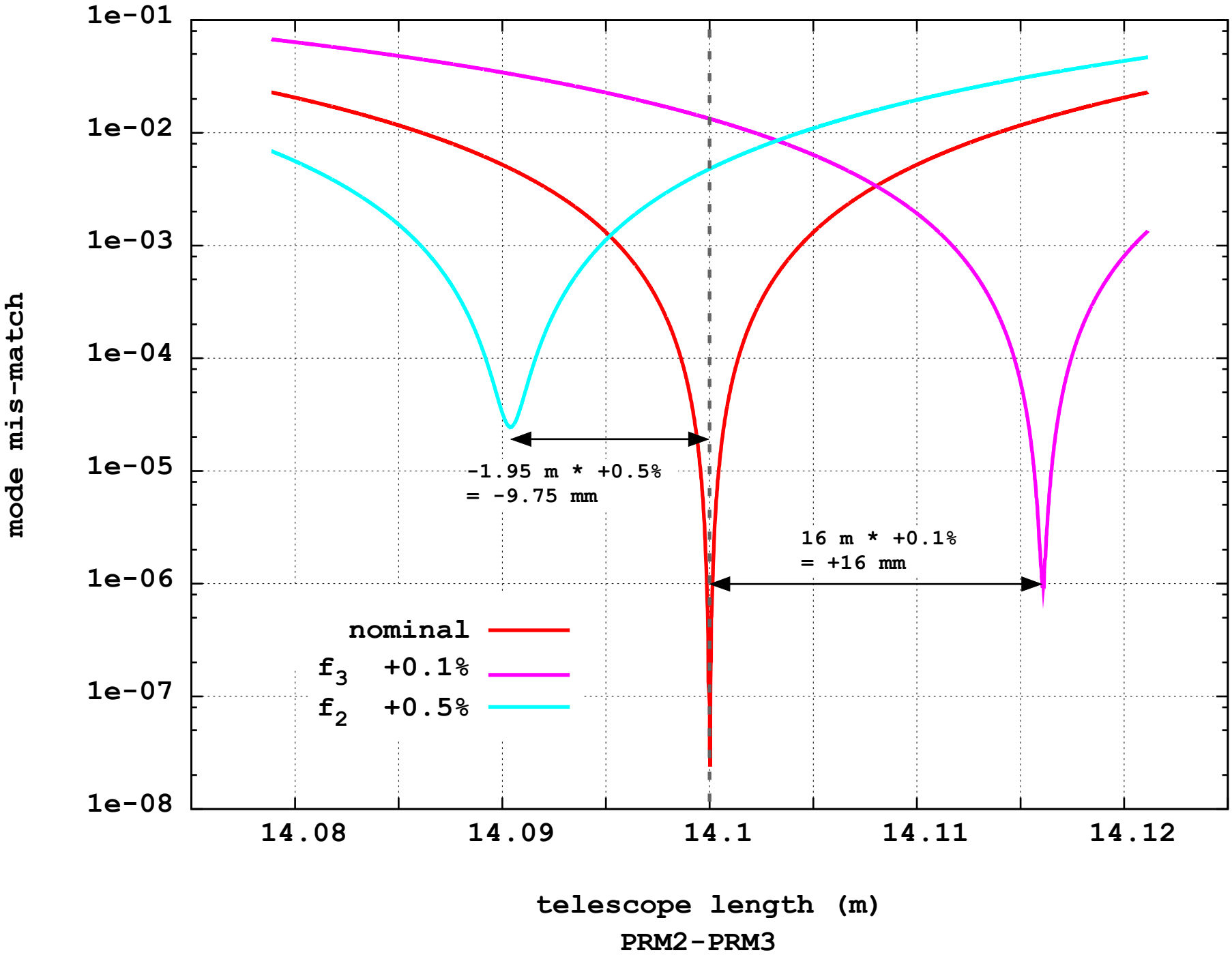
$$\begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/f_3 & 1 \end{pmatrix} = \begin{pmatrix} \frac{f_2-d}{f_2} & d \\ -1/F & \frac{f_3-d}{f_3} \end{pmatrix}$$

$$F = \frac{f_1 f_2}{f_1 + f_2 - d}$$

f_1=-1.95, f_2=+16.0, d=14.1

F = -624

f_1+f_2-d is the most sensitive parameter.


1e-01
1e-02
1e-03
1e-04
1e-05
1e-06
1e-07
1e-08
mode mis-match
14.08
14.09
14.1
14.11
14.12
telescope length (m)
PRM2-PRM3
-1.95 m * +0.5%
= -9.75 mm
16 m * +0.1%
= +16 mm
nominal
f3 +0.1%
f2 +0.5%

Summary of ROC error assessment

		mode mis-match with 1% ROC error	Comments
1. Arm cavity (g_1=g_2=+0.5783)		1e-5	common, differetial で差はない
2. Arm cavity (g_1=g_2=-0.5783)		1e-4	
3. No-lens PRC design	positive-g	1e-2	g_1=g_2=+0.5783
	negative-g	**X**	g_1=g_2= -0.5783
4. Folded PRC with g_1=g_2=+0.5783	ITM	1e-5	
	PRM	1e-6	
	telescope	**X**	**0.05% error + telescope 調整**
~~**5. Folded PRC** with g_1=g_2= -0.5783~~			