Design of dielectric mulilayer coating

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Of the many techniques to express the refectance and transmittance of the multilayer coating mirror, the 'Matrix-method' can express the characteristic matrix of this LH pair film as follows,

$$M = \begin{pmatrix} \cos\delta_L & \dfrac{i\,\sin\delta_L}{\eta_L} \\ i\,\eta_L\sin\delta_L & \cos\delta_L \end{pmatrix} \begin{pmatrix} \cos\delta_H & \dfrac{i\,\sin\delta_H}{\eta_H} \\ i\,\eta_H\sin\delta_H & \cos\delta_H \end{pmatrix} \tag{1}$$

where $\eta_{\mathrm{i=0,\ L,\ H}}$ and $\delta_{\mathrm{i=0,\ L,\ H}}$ are the complex effective refractive index and the phase thickness of the incident medium, the L film and the H film, respedively. The effective refractive index($\eta_{\mathrm{i=0,\ L,\ H}}$) is defined as follows for S and P polarization,

$$\eta_{\mathrm{i}} = n_{\mathrm{i}}\cos\phi_{\mathrm{i}} \qquad S \text{ polarization} \tag{2}$$

$$= \frac{\eta_{\mathrm{i}}}{\cos\phi_{\mathrm{i}}} \qquad P \text{ polarization,} \tag{3}$$

where $n_{\mathrm{i=0,\ L,\ H}}$ are the complex refractive index and the incident angle for the incident medium, the L film and the H film, respectively. Assuming the optical thickness($n_i\,d_i$) to be $\lambda/4$, the phase thickness $\delta_{\mathrm{i=0,\ L,\ H}}$ can be simplified to

$$\delta_{\mathrm{i}} = \frac{2\pi n_i d_i}{\lambda}\cos\phi_i \tag{4}$$

$$\rightarrow \frac{\pi}{2}\cos\phi_i\,, \tag{5}$$

where $d_{\mathrm{i=0,\ L,\ H}}$ is the physical thickness.

The matrix (C) of the dielectric multilayer coating mirror which has, for example, N LH pair films and one H film on its top is expressed as follows

$$C = \begin{pmatrix} \cos\delta_H & \dfrac{i\,\sin\delta_H}{\eta_H} \\ i\,\eta_H\sin\delta_H & \cos\delta_H \end{pmatrix} {M_{\mathrm{LH}}}^N \tag{6}$$

$$= \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \tag{7}$$

Theoretically, the power reflectance and the power transmittance of the mirror whose chatracteristic matrix is expressed as C can be calculated as follows,

$$R = \left| \frac{\eta_0(c_{11} + \eta_S\, c_{12}) - (c_{21} + \eta_S\, c_{22})}{\eta_0(c_{11} + \eta_S\, c_{12}) + (c_{21} + \eta_S\, c_{22})} \right|^2 \tag{8}$$

$$T = \frac{\eta_S}{\eta_0} \left| \frac{2\,\eta_0}{\eta_0(c_{11} + \eta_S\, c_{12}) + (c_{21} + \eta_S\, c_{22})} \right|^2 , \tag{9}$$

where subscript 'S' means the substrate of the mirror. Consequently, the reflectance and the transmittance of the mirror can be decided by four parameters: the repfractive indexes of the H and L films ($n_{\rm L}, n_{\rm H}$), the layered number of the LH pair films (N) and the incident beam angle (ϕ_0). Practically, the only unknown parameters are $n_{\rm L}$ and $n_{\rm H}$ because N is a known number from its manufacturing process.

Technique to Estimate the Reflectance of a High-Reflectance Dielectric Multilayer Coating Mirror Using Incident Beam Angular Dependence of Its Transmittance.

参考文献

[1] S.Miyoki, S.Sato, M.Ohashi and M.-K. Fujimoto, Optical Review, **5**, (1998), 17-19

[2] 4) R.E.Hummel and K.H.Guenther, Handbook of *OPICAL PROPERTTES* 'Thin Films for Optical Coating' (CRC Press, USA, 1995) Vol.1, Chap.4, p.79